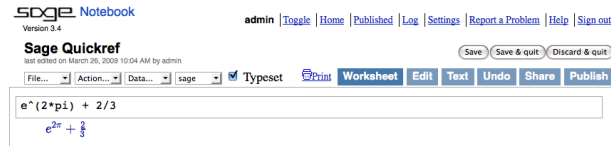


Sage Quick Reference

William Stein (based on work of P. Jipsen) (mod. by nu)
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Notebook Notebook



セルの評価: `<shift-enter>`

セルを評価し新しいセルを作る: `<alt-enter>`

セルの分割: `<control-; >`

セルの結合: `<control-backspace>`

数式セルの挿入: セルの間の青い線をクリック

Text/HTML セルの挿入: セルの間の青い線を shift-click

セルの削除: 内容を削除したあとで backspace

- ORIGINAL TEXT
- Evaluate cell: `<shift-enter>`
- Evaluate cell creating new cell: `<alt-enter>`
- Split cell: `<control-; >`
- Join cells: `<control-backspace>`
- Insert math cell: click blue line between cells
- Insert text/HTML cell: shift-click blue line between cells
- Delete cell: delete content then backspace

コマンドライン Command line

`com<tab>` で `command` を補完

`*bar*?` で “bar” を含むコマンド名をリストアップ

`command?<tab>` でドキュメントを表示

`command??<tab>` でソースコードを表示

`a.<tab>` でオブジェクト a のメソッドを表示 (`dir(a)` も)

`a._<tab>` で a の hidden methods を表示

`search_doc("string or regexp")` ドキュメントの全文検索

`search_src("string or regexp")` ソースコードの検索

`_` は直前の出力

- ORIGINAL TEXT
- `com<tab>` complete `command`
- `*bar*?` list command names containing “bar”
- `command?<tab>` shows documentation
- `command??<tab>` shows source code
- `a.<tab>` shows methods for object a (more: `dir(a)`)
- `a._<tab>` shows hidden methods for object a
- `search_doc("string or regexp")` fulltext search of docs
- `search_src("string or regexp")` search source code
- `_` is previous output

数 Numbers

整数: $\mathbb{Z} = \mathbb{ZZ}$ 例 `-2 -1 0 1 10^100`

有理数: $\mathbb{Q} = \mathbb{QQ}$ 例 `1/2 1/1000 314/100 -2/1`

実数: $\mathbb{R} \approx \mathbb{RR}$ 例 `.5 0.001 3.14 1.23e10000`

複素数: $\mathbb{C} \approx \mathbb{CC}$ 例 `CC(1,1) CC(2.5,-3)`

倍精度 (Double): RDF and CDF 例 `CDF(2.1,3)`

Mod n : $\mathbb{Z}/n\mathbb{Z} = \mathbb{Zmod}$ 例 `Mod(2,3) Zmod(3)(2)`

有限体: $\mathbb{F}_q = \mathbb{GF}$ 例 `GF(3)(2) GF(9, "a").0`

多項式: $R[x, y]$ 例 `S.<x,y>=QQ[] x+2*y^3`

巾級数: $R[[t]]$ 例 `S.<t>=QQ[] 1/2+2*t+0(t^2)`

p 進整数: $\mathbb{Z}_p \approx \mathbb{Zp}$, $\mathbb{Q}_p \approx \mathbb{Qp}$ 例 `2+3*5+0(5^2)`

代数閉包: $\overline{\mathbb{Q}} = \mathbb{QQbar}$ 例 `QQbar(2^(1/5))`

区間演算: RIF 例 `RIF((1,1.00001))`

数体: `R.<x>=QQ[]`; `K.<a>=NumberField(x^3+x+1)`

- ORIGINAL TEXT
- Integers: $\mathbb{Z} = \mathbb{ZZ}$ e.g. `-2 -1 0 1 10^100`
- Rationals: $\mathbb{Q} = \mathbb{QQ}$ e.g. `1/2 1/1000 314/100 -2/1`
- Reals: $\mathbb{R} \approx \mathbb{RR}$ e.g. `.5 0.001 3.14 1.23e10000`
- Complex: $\mathbb{C} \approx \mathbb{CC}$ e.g. `CC(1,1) CC(2.5,-3)`
- Double precision: RDF and CDF e.g. `CDF(2.1,3)`
- Mod n : $\mathbb{Z}/n\mathbb{Z} = \mathbb{Zmod}$ e.g. `Mod(2,3) Zmod(3)(2)`
- Finite fields: $\mathbb{F}_q = \mathbb{GF}$ e.g. `GF(3)(2) GF(9, "a").0`
- Polynomials: $R[x, y]$ e.g. `S.<x,y>=QQ[] x+2*y^3`
- Series: $R[[t]]$ e.g. `S.<t>=QQ[] 1/2+2*t+0(t^2)`
- p -adic numbers: $\mathbb{Z}_p \approx \mathbb{Zp}$, $\mathbb{Q}_p \approx \mathbb{Qp}$ e.g. `2+3*5+0(5^2)`
- Algebraic closure: $\overline{\mathbb{Q}} = \mathbb{QQbar}$ e.g. `QQbar(2^(1/5))`
- Interval arithmetic: RIF e.g. `RIF((1,1.00001))`
- Number field: `R.<x>=QQ[]`; `K.<a>=NumberField(x^3+x+1)`

四則演算など Arithmetic

$ab = a*b$ $\frac{a}{b} = a/b$ $a^b = a^b$ $\sqrt{x} = \text{sqrt}(x)$

$\sqrt[n]{x} = x^(1/n)$ $|x| = \text{abs}(x)$ $\log_b(x) = \text{log}(x,b)$

和: $\sum_{i=k}^n f(i) = \text{sum}(f(i) \text{ for } i \text{ in } (k..n))$

積: $\prod_{i=k}^n f(i) = \text{prod}(f(i) \text{ for } i \text{ in } (k..n))$

- ORIGINAL TEXT
- $ab = a*b$ $\frac{a}{b} = a/b$ $a^b = a^b$ $\sqrt{x} = \text{sqrt}(x)$
- $\sqrt[n]{x} = x^(1/n)$ $|x| = \text{abs}(x)$ $\log_b(x) = \text{log}(x,b)$
- Sums: $\sum_{i=k}^n f(i) = \text{sum}(f(i) \text{ for } i \text{ in } (k..n))$
- Products: $\prod_{i=k}^n f(i) = \text{prod}(f(i) \text{ for } i \text{ in } (k..n))$

定数と関数 Constants and functions

定数: $\pi = \text{pi}$ $e = \text{e}$ $i = \text{i}$ $\infty = \text{oo}$

$\phi = \text{golden_ratio}$ $\gamma = \text{euler_gamma}$

近似値: `pi.n(digits=18) = 3.14159265358979324`

関数: `sin cos tan sec csc cot sinh cosh tanh sech csch coth log ln exp ...`

Python の関数: `def f(x): return x^2`

- ORIGINAL TEXT
- Constants: $\pi = \text{pi}$ $e = \text{e}$ $i = \text{i}$ $\infty = \text{oo}$
- $\phi = \text{golden_ratio}$ $\gamma = \text{euler_gamma}$
- Approximate: `pi.n(digits=18) = 3.14159265358979324`
- Functions: `sin cos tan sec csc cot sinh cosh tanh sech csch coth log ln exp ...`
- Python function: `def f(x): return x^2`

インタラクティブな操作 Interactive functions

関数の前に `@interact` を置く (変数で controls が決まる)

```
@interact
def f(n=[0..4], s=(1..5), c=Color("red")):
    var("x")
    show(plot(sin(n+x*s), -pi, pi, color=c))
```

- ORIGINAL TEXT
- Put `@interact` before function (vars determine controls)
- `@interact`
- `def f(n=[0..4], s=(1..5), c=Color("red")):`
- `var("x")`
- `show(plot(sin(n+x*s), -pi, pi, color=c))`

シンボリックな数式 Symbolic expressions

新しい不定元 (symbolic variables) を定義: `var("t u v y z")`

シンボリックな関数 (Symbolic function):

例 $f(x) = x^2$ `f(x)=x^2`

関係式: `f==g f<=g f>=g f<g f>g`

$f = g$ を解く: `solve(f(x)==g(x), x)`
`solve([f(x,y)==0, g(x,y)==0], x,y)`

`factor(...)` `expand(...)` `(...).simplify_...`

$x \in [a, b]$ s.t. $f(x) \approx 0$ を見付ける: `find_root(f(x), a, b)`

- ORIGINAL TEXT
- Define new symbolic variables: `var("t u v y z")`
- Symbolic function: e.g. $f(x) = x^2$ `f(x)=x^2`
- Relations: `f==g f<=g f>=g f<g f>g`
- Solve $f = g$: `solve(f(x)==g(x), x)`
- `solve([f(x,y)==0, g(x,y)==0], x,y)`
- `factor(...)` `expand(...)` `(...).simplify_...`
- `find_root(f(x), a, b)` find $x \in [a, b]$ s.t. $f(x) \approx 0$

微分積分 Calculus

$\lim_{x \rightarrow a} f(x) = \text{limit}(f(x), x=a)$

$\frac{d}{dx} f(x) = \text{diff}(f(x), x)$

$\frac{\partial}{\partial x} f(x, y) = \text{diff}(f(x, y), x)$

`diff = differentiate = derivative`

$\int f(x) dx = \text{integral}(f(x), x)$

$\int_a^b f(x) dx = \text{integral}(f(x), x, a, b)$

$\int_a^b f(x) dx \approx \text{numerical_integral}(f(x), a, b)$

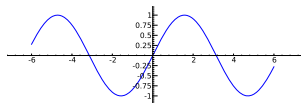
a に関する次数 n の Taylor 多項式: `taylor(f(x), x, a, n)`

```

..... ORIGINAL TEXT
lim_{x \to a} f(x) = limit(f(x), x=a)
\frac{d}{dx}(f(x)) = diff(f(x), x)
\frac{\partial}{\partial x}(f(x, y)) = diff(f(x, y), x)
diff = differentiate = derivative
\int f(x) dx = integral(f(x), x)
\int_a^b f(x) dx = integral(f(x), x, a, b)
\int_a^b f(x) dx \approx numerical\_integral(f(x), a, b)
Taylor polynomial, deg n about a: taylor(f(x), x, a, n)

```

二次元グラフィックス 2D graphics



```

line([(x1, y1), ..., (xn, yn)], options)
polygon([(x1, y1), ..., (xn, yn)], options)
circle((x, y), r, options)
text("txt", (x, y), options)

```

options は `plot.options` にあるものを使用、
例 `thickness=pixel`, `rgbcolor=(r,g,b)`, `hue=h`
ただし $0 \leq r, b, g, h \leq 1$

`show(graphic, options)`

サイズの調整には `figsize=[w,h]` を使う

縦横比を調整するには `aspect_ratio=number` を使う

```

plot(f(x), (x, xmin, xmax), options)
parametric_plot((f(t), g(t)), (t, tmin, tmax), options)
polar_plot(f(t), (t, tmin, tmax), options)
結合: circle((1, 1), 1)+line([(0, 0), (2, 2)])
animate(list of graphics, options).show(delay=20)

```

```

..... ORIGINAL TEXT
line([(x1, y1), ..., (xn, yn)], options)
polygon([(x1, y1), ..., (xn, yn)], options)
circle((x, y), r, options)
text("txt", (x, y), options)
options as in plot.options,
e.g. thickness=pixel, rgbcolor=(r,g,b), hue=h
where 0 \le r, b, g, h \le 1
show(graphic, options)

```

use `figsize=[w,h]` to adjust size

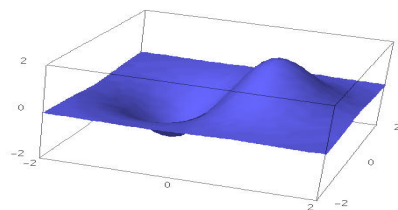
use `aspect_ratio=number` to adjust aspect ratio

```

plot(f(x), (x, xmin, xmax), options)
parametric_plot((f(t), g(t)), (t, tmin, tmax), options)
polar_plot(f(t), (t, tmin, tmax), options)
combine: circle((1, 1), 1)+line([(0, 0), (2, 2)])
animate(list of graphics, options).show(delay=20)

```

三次元グラフィックス 3D graphics



```

line3d([(x1, y1, z1), ..., (xn, yn, zn)], options)
sphere((x, y, z), r, options)
text3d("txt", (x, y, z), options)
tetrahedron((x, y, z), size, options)
cube((x, y, z), size, options)
octahedron((x, y, z), size, options)
dodecahedron((x, y, z), size, options)
icosahedron((x, y, z), size, options)
plot3d(f(x, y), (x, xb, xe), (y, yb, ye), options)
parametric_plot3d((f, g, h), (t, tb, te), options)
parametric_plot3d((f(u, v), g(u, v), h(u, v)),
                  (u, ub, ue), (v, vb, ve), options)
options: aspect_ratio=[1, 1, 1], color="red",
          opacity=0.5, figsize=6, viewer="tachyon"

```

```

..... ORIGINAL TEXT
line3d([(x1, y1, z1), ..., (xn, yn, zn)], options)
sphere((x, y, z), r, options)
text3d("txt", (x, y, z), options)
tetrahedron((x, y, z), size, options)
cube((x, y, z), size, options)
octahedron((x, y, z), size, options)
dodecahedron((x, y, z), size, options)
icosahedron((x, y, z), size, options)
plot3d(f(x, y), (x, xb, xe), (y, yb, ye), options)
parametric_plot3d((f, g, h), (t, tb, te), options)
parametric_plot3d((f(u, v), g(u, v), h(u, v)),
                  (u, ub, ue), (v, vb, ve), options)
options: aspect_ratio=[1, 1, 1], color="red",
          opacity=0.5, figsize=6, viewer="tachyon"

```

離散数学 Discrete math

$\lfloor x \rfloor = \text{floor}(x)$ $\lceil x \rceil = \text{ceil}(x)$

n を k で割った余り = $n\%k$ $k|n$ iff $n\%k==0$

$n! = \text{factorial}(n)$ $\binom{x}{m} = \text{binomial}(x, m)$

$\phi(n) = \text{euler_phi}(n)$

文字列 (String): 例 `s = "Hello" = "He"+"llo"`

`s[0]="H" s[-1]="o" s[1:3]="el" s[3:]="lo"`

リスト (List): 例 `[1, "Hello", x] = []+[1, "Hello"]+[x]`

タプル (Tuple): 例 `(1, "Hello", x)` (immutable)

集合 (Set): 例 `{1, 2, 1, a} = Set([1, 2, 1, "a"]) (= {1, 2, a})`

集合の内包的記法 \approx リストの内包表記, 例

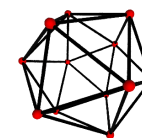
`{f(x)|x \in X, x > 0} = Set([f(x) for x in X if x > 0])`

```

..... ORIGINAL TEXT
[x] = floor(x)   [x] = ceil(x)
Remainder of n divided by k = n%k   k|n iff n%k==0
n! = factorial(n)   \binom{x}{m} = binomial(x, m)
phi(n) = euler_phi(n)
Strings: e.g. s = "Hello" = "He"+"llo"
s[0]="H"   s[-1]="o"   s[1:3]="el"   s[3:]="lo"
Lists: e.g. [1, "Hello", x] = []+[1, "Hello"]+[x]
Tuples: e.g. (1, "Hello", x) (immutable)
Sets: e.g. {1, 2, 1, a} = Set([1, 2, 1, "a"]) (= {1, 2, a})
List comprehension \approx set builder notation, e.g.
{f(x)|x \in X, x > 0} = Set([f(x) for x in X if x > 0])

```

グラフ理論 Graph theory



グラフ: `G = Graph({0:[1,2,3], 2:[4]})`

有向グラフ: `DiGraph(dictionary)`

グラフの族: `graphs.<tab>`

不変量: `G.chromatic_polynomial()`, `G.is_planar()`

パス: `G.shortest_path()`

可視化: `G.plot()`, `G.plot3d()`

自己同型: `G.automorphism_group()`,

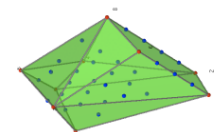
`G1.is_isomorphic(G2)`, `G1.is_subgraph(G2)`

```

..... ORIGINAL TEXT
Graph: G = Graph({0:[1,2,3], 2:[4]})
Directed Graph: DiGraph(dictionary)
Graph families: graphs.<tab>
Invariants: G.chromatic_polynomial(), G.is_planar()
Paths: G.shortest_path()
Visualize: G.plot(), G.plot3d()
Automorphisms: G.automorphism_group(),
G1.is_isomorphic(G2), G1.is_subgraph(G2)

```

組合せ論 Combinatorics



整数列: `sloane_find(list)`, `sloane.<tab>`

分割: `P=Partitions(n)` `P.count()`

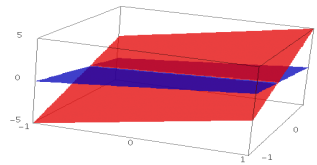
組合せ (部分リスト): `C=Combinations(list)` `C.list()`

直積: `CartesianProduct(P, C)`

ヤング盤 (Tableau): `Tableau([[1,2,3],[4,5]])`
ワード: `W=Words("abc"); W("aabca")`
半順序集合 (poset): `Poset([[1,2],[4],[3],[4],[]])`
ルート系: `RootSystem(["A",3])`
クリスタル: `CrystalOfTableaux(["A",3], shape=[3,2])`
格子多面体: `A=random_matrix(ZZ,3,6,x=7)`
`L=LatticePolytope(A) L.npoints() L.plot3d()`

$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \text{matrix}(\text{QQ}, 2, 3, [1, 2, 3, 4, 5, 6])$
 $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \det(\text{matrix}(\text{QQ}, [[1, 2], [3, 4]]))$
 $Av = A*v \quad A^{-1} = A^{-1} \quad A^t = A.\text{transpose}()$
Solve $Ax = v$: `A\v` or `A.solve_right(v)`
Solve $xA = v$: `A.solve_left(v)`
Reduced row echelon form: `A.echelon_form()`
Rank and nullity: `A.rank()` `A.nullity()`
Hessenberg form: `A.hessenberg_form()`
Characteristic polynomial: `A.charpoly()`
Eigenvalues: `A.eigenvalues()`
Eigenvectors: `A.eigenvectors_right()` (also left)
Gram-Schmidt: `A.gram_schmidt()`
Visualize: `A.plot()`
LLL reduction: `matrix(ZZ, ...).LLL()`
Hermite form: `matrix(ZZ, ...).hermite_form()`

線形代数 Linear algebra



ベクトル空間 $K^n = K^n$ 例 `QQ^3 RR^2 CC^4`

部分空間: `span(vectors, field)`
例 `span([[1,2,3], [2,3,5]], QQ)`

Kernel: `A.right_kernel()` (left_ も)
和と共通部分: `V + W` と `V.intersection(W)`

基底: `V.basis()`

基底行列: `V.basis_matrix()`

行列を部分空間への制限: `A.restrict(V)`

基底を使ったベクトルの表示: `V.coordinates(vector)`

Vector space $K^n = K^n$ e.g. `QQ^3 RR^2 CC^4`
Subspace: `span(vectors, field)`
E.g., `span([[1,2,3], [2,3,5]], QQ)`
Kernel: `A.right_kernel()` (also left)
Sum and intersection: `V + W` and `V.intersection(W)`
Basis: `V.basis()`
Basis matrix: `V.basis_matrix()`
Restrict matrix to subspace: `A.restrict(V)`
Vector in terms of basis: `V.coordinates(vector)`

数値計算 Numerical mathematics

パッケージ: `import numpy, scipy, cvxopt`

最小化: `var("x y z")`

`minimize(x^2+x*y^3+(1-z)^2-1, [1,1,1])`

Packages: `import numpy, scipy, cvxopt`
Minimization: `var("x y z")`

`minimize(x^2+x*y^3+(1-z)^2-1, [1,1,1])`

整数論 Number theory

素数: `prime_range(n,m)`, `is_prime`, `next_prime`

素因数分解: `factor(n)`, `qsieve(n)`, `ecm.factor(n)`

Kronecker symbol: $\left(\frac{a}{b}\right) = \text{kronecker_symbol}(a,b)$

連分数: `continued_fraction(x)`

Bernoulli 数: `bernoulli(n)`, `bernoulli_mod_p(p)`

楕円曲線: `EllipticCurve([a1, a2, a3, a4, a6])`

Dirichlet characters: `DirichletGroup(N)`

Modular forms: `ModularForms(level, weight)`

Modular symbols: `ModularSymbols(level, weight, sign)`

Brandt modules: `BrandtModule(level, weight)`

Modular abelian varieties: `J0(N)`, `J1(N)`

Primes: `prime_range(n,m)`, `is_prime`, `next_prime`
Factor: `factor(n)`, `qsieve(n)`, `ecm.factor(n)`
Kronecker symbol: $\left(\frac{a}{b}\right) = \text{kronecker_symbol}(a,b)$
Continued fractions: `continued_fraction(x)`
Bernoulli numbers: `bernoulli(n)`, `bernoulli_mod_p(p)`
Elliptic curves: `EllipticCurve([a1, a2, a3, a4, a6])`
Dirichlet characters: `DirichletGroup(N)`
Modular forms: `ModularForms(level, weight)`
Modular symbols: `ModularSymbols(level, weight, sign)`
Brandt modules: `BrandtModule(level, weight)`
Modular abelian varieties: `J0(N)`, `J1(N)`

群論 Group theory

`G = PermutationGroup([(1,2,3), (4,5)], [(3,4)])`

`SymmetricGroup(n)`, `AlternatingGroup(n)`

アーベル群: `AbelianGroup([3,15])`

行列群: `GL`, `SL`, `Sp`, `SU`, `GU`, `SO`, `GO`

関数: `G.sylow_subgroup(p)`, `G.character_table()`,
`G.normal_subgroups()`, `G.cayley_graph()`

`G = PermutationGroup([(1,2,3), (4,5)], [(3,4)])`
`SymmetricGroup(n)`, `AlternatingGroup(n)`
Abelian groups: `AbelianGroup([3,15])`
Matrix groups: `GL`, `SL`, `Sp`, `SU`, `GU`, `SO`, `GO`
Functions: `G.sylow_subgroup(p)`, `G.character_table()`,
`G.normal_subgroups()`, `G.cayley_graph()`

非可換環 Noncommutative rings

四元数: `Q.<i,j,k> = QuaternionAlgebra(a,b)`

自由代数: `R.<a,b,c> = FreeAlgebra(QQ, 3)`

Quaternions: `Q.<i,j,k> = QuaternionAlgebra(a,b)`
Free algebra: `R.<a,b,c> = FreeAlgebra(QQ, 3)`

Python のモジュール Python modules

行列代数 Matrix algebra

$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \text{vector}([1,2])$

$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \text{matrix}(\text{QQ}, [[1,2],[3,4]], \text{sparse=False})$

$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \text{matrix}(\text{QQ}, 2, 3, [1, 2, 3, 4, 5, 6])$

$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = \det(\text{matrix}(\text{QQ}, [[1,2],[3,4]]))$

$Av = A*v \quad A^{-1} = A^{-1} \quad A^t = A.\text{transpose}()$

$Ax = v$ を解く: `A\v` or `A.solve_right(v)`

$xA = v$ を解く: `A.solve_left(v)`

被約行階段行列: `A.echelon_form()`

階数と退化: `A.rank()` `A.nullity()`

Hessenberg 型: `A.hessenberg_form()`

特性多項式: `A.charpoly()`

固有値: `A.eigenvalues()`

固有ベクトル: `A.eigenvectors_right()` (also left)

Gram-Schmidt: `A.gram_schmidt()`

可視化: `A.plot()`

LLL reduction: `matrix(ZZ, ...).LLL()`

Hermite 形式: `matrix(ZZ, ...).hermite_form()`

$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \text{vector}([1,2])$

$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \text{matrix}(\text{QQ}, [[1,2],[3,4]], \text{sparse=False})$

```
import module_name
module_name.<tab> and help(module_name)
..... ORIGINAL TEXT
import module_name
module_name.<tab> and help(module_name)
```

解析とデバッグ Profiling and debugging

time *command*: timing information の表示

timeit("command"): accurately time command

t = cputime(); **cputime(t)**: 経過した CPU time

t = walltime(); **walltime(t)**: 経過した wall time

%pdb: interactive debugger を開始 (command line only)

%prun *command*: profile command (command line only)

```
..... ORIGINAL TEXT
time command: show timing information
timeit("command"): accurately time command
t = cputime(); cputime(t): elapsed CPU time
t = walltime(); walltime(t): elapsed wall time
%pdb: turn on interactive debugger (command line only)
%prun command: profile command (command line only)
```