

Sage Dynamics Ref Card v3.0

(for Sage 8.1)

Rings and Fields

ZZ	integer ring	Zmod(<i>m</i>)	$\mathbb{Z}/m\mathbb{Z}$
QQ	rational field	QQbar	alg. clos. of QQ
RR	real field	CC	complex field
Qp(<i>p</i>)	<i>p</i> -adic field	Zp(<i>p</i>)	<i>p</i> -adic integers
QQ[]	polynomials	QQ[][]	power series
GF(<i>p</i>)	prime field	GF(<i>p</i> ^{<i>n</i>} , ' <i>v</i> ')	finite field
CyclotomicField(<i>n</i>)		Q(ζ _{<i>n</i>})	
FractionField(<i>ring</i>)			field of fractions
QuadraticField(<i>d</i>)		Q(√ <i>d</i>)	
NumberField(<i>poly</i> , ' <i>var</i> ', [<i>emb</i>])			number field
K.absolute_field()			—
K.degree()		[<i>K</i> : Q]	
K.extension(<i>poly</i>)			fld ext
QQ.range_by_height(<i>bd</i>)			iterator
K.elements_of_bounded_height(<i>bd</i> , [<i>params</i>])			
number_field_elements_from_algebraics(<i>pts</i>)			

Spaces and Schemes

A.< <i>vars</i> >=AffineSpace(<i>ring</i> , <i>dim</i>)		\mathbb{A}^n	
P.< <i>vars</i> >=ProjectiveSpace(<i>ring</i> , <i>dim</i>)		\mathbb{P}^n	
PP.< <i>vars</i> >=ProductProjectiveSpaces(<i>ring</i> , <i>dims</i>)		$\mathbb{P}^n \times \dots \times \mathbb{P}^m$	
WehlerK3Surface(<i>polys</i>)			—
S.affine_patch(<i>i</i> , [A])			—
S.base_ring()			base ring S
S.change_ring()			change base ring
S.coordinate_ring()			coord. ring of S
S.defining_ideal()			—
S.defining_polynomials()			—
S.dimension()			rel. dim of S
S.gens()			vars of coord. ring
S.point_transformation_matrix([<i>pts</i> , <i>pts</i>])			find PGL element
S.projective_embedding([<i>i</i> , P])			—
S.projective_closure([<i>i</i> , P])			—
S.rational_points([<i>bd</i> , fld])			—
S.subscheme(<i>polys</i>)			subscheme of S
S.vars()			vars of coord. ring
S.variable_names()			vars as strings
S.weil_restriction()			restric. of const.

Dynamical System Initialization

DynamicalSystem(<i>polys</i> , [<i>domain</i>])			projective if no domain
DynamicalSystem.affine(<i>polys</i> , [<i>domain</i>])			
DynamicalSystem.projective(<i>polys</i> , [<i>domain</i>])			
f.as_dynamical_system()		End	→ DS

Periodic Behavior

f.dynatomic_polynomial([<i>m</i> , <i>n</i>])		—	
Q.is_preperiodic(<i>f</i>)		—	
Q.multiplier(<i>f</i> , <i>n</i>)		(<i>f</i> ^{<i>n</i>})'(Q)	
Q.orbit_structure(<i>f</i>)		[tail, period]	
f.periodic_points(<i>n</i> , [<i>params</i>])		—	
f.rational_periodic_points([<i>params</i>])		—	
f.rational_periodic_graph([<i>params</i>])		—	
f.rational_preperiodic_points([<i>params</i>])		—	
f.rational_preperiodic_graph([<i>params</i>])		—	
f.possible_periods([<i>params</i>])		via good red.	

Heights and Measures

Q.canonical_height(<i>f</i> , [<i>params</i>])		$\hat{h}_f(Q)$	
f.critical_height()		$\sum_{c \in \text{Crit}} \hat{h}_f(c)$	
Q.global_height([<i>prec</i>])		$h(Q)$	
f.global_height([<i>prec</i>])		—	
Q.green_function(<i>v</i> , [<i>prec</i>])		at <i>v</i>	
f.height_difference_bound()		$ h(Q) - \hat{h}_f(Q) $	
f.local_height_arch(<i>i</i> , [<i>prec</i>])		at ∞	

Critical Points

f.critical_points()		—	
f.critical_subscheme()		—	
f.critical_point_portrait()		—	
f.critical_height()		$\sum_{c \in \text{Crit}} \hat{h}_f(c)$	
f.is_postcritically_finite()		—	
f.wronskian_ideal()		crit locus	

Cyclic Structures

f.all_rational_preimages(<i>points</i>)		—	
f.cyclegraph()		\mathbb{F}_q	digraph
Q.orbit_structure(<i>f</i>)		\mathbb{F}_q	[tail, per]
Q.rational_preimages(<i>f</i>)		—	
Q.rational_connected_component(<i>f</i>)		—	

Rational Functions

f.dynamical_degree()		—	
f.degree_sequence()			deg. of iterates
f.indeterminacy_locus()		—	
f.indeterminacy_points()			if fin. many

Functions

f[<i>i</i>]			<i>i</i> th coord
f.automorphism_group()			{ $\phi : f^\phi = f$ }
f.autmorphism_group()			Hom(<i>f</i> , <i>f</i>)
f.base_ring()			—
f.change_ring()			—
P.chebyshev_polynomial(<i>k</i> , <i>kind</i>)			
f.codomain()			—
f.conjugate(ϕ)			$\phi^{-1} \circ f \circ \phi$
f.conjugating_set(<i>g</i>)			Hom(<i>f</i> , <i>g</i>)
f.defining_polynomials()			—
f.degree()			—
f.dehomogenize(<i>k</i>)			—
f.domain()			—
f.homogenize(<i>k</i>)			—
f.is_morphism()			—
f.normalize_coordinates()			remove gcd
f.nth_iterate(Q, <i>n</i>)			<i>f</i> ^{<i>n</i>} (Q)
f.nth_iterate_map(<i>n</i>)			<i>f</i> ^{<i>n</i>}
P.Lattes(E, <i>m</i>)			create Lattès map
f.orbit(Q, [<i>m</i> , <i>n</i>])			{ <i>f</i> ^{<i>m</i>} (Q), ..., <i>f</i> ^{<i>n</i>} (Q)}
f.primes_of_bad_reduction()			—
f.resultant()			—
f.scale_by(<i>t</i>)			<i>t</i> · <i>f</i>
f.specialization()			subs value of param

Points

Q[<i>i</i>]			<i>i</i> th coord
Q.change_ring()			—
Q.clear_denominator()			—
Q.codomain()			ambient space
Q.dehomogenize(<i>i</i>)			—
Q.domain()			base ring
Q.homogenize(<i>i</i>)			—
Q.normalize_coordinates()			remove gcd
Q.nth_iterate(<i>f</i> , <i>n</i>)			<i>f</i> ^{<i>n</i>} (Q)
Q.orbit(<i>f</i> , (<i>m</i> , <i>n</i>))			[<i>f</i> ^{<i>m</i>} (Q), ..., <i>f</i> ^{<i>n</i>} (Q)]
Q.scale_by(<i>t</i>)			<i>t</i> · Q

Iteration

f.nth_iterate(Q, n)	$f^n(Q)$
f.nth_iterate_map(n)	f^n
f.orbit($Q, [m, n]$)	$[f^m(Q), \dots, f^n(Q)]$
f.rational_preimages(Q, k)	$f^{-k}(Q)$

Moduli Spaces

f.is_polynomial()	has tot. ram. fixed pt.
f.is_PGL_minimal()	
f.is_conjugate(g)	$g \stackrel{?}{=} f^\phi$
f.normal_form()	$x^n + a_{n-2}x^{n-2} + \dots + a_0$
f.minimal_model()	min resultant f^ϕ
f.multiplier_spectra($n, [params]$)	$\{\lambda_f(Q) : Q \in \text{Per}_n\}$
f.sigma_invariants($n, [params]$)	$\{\sigma_i(\lambda_f(Q)) : Q \in \text{Per}_n\}$

Finite Fields

f.cyclegraph()	iteration digraph
Q.orbit_structure(f)	[tail, period]

Mandelbrot and Julia Sets

external_ray(v)	list or single angle
mandelbrot_plot([params])	for $z^2 + c$
julia_plot([params])	for $z^2 + c$

Miscellaneous / Help

-	last output
%time	execution time
timeit('cmd', number=#)	time multiple iterations
s.<tab>	show all cmds on s
s.cmd?	info about cmd on s
set_verbosity(None)	disable warnings
load('path to file')	load code file
copy(obj)	—
latex(obj)	—
all(list of bool)	—
any(list of bool)	—
sum(list)	—
max(list)	—
isinstance(f, type)	check for type
preparser(bool)	on/off notebk preparing

Matrices

matrix(K, n, m, list)	create matrix
matrix(K, list of lists)	create matrix
M.charpoly()	—
M.determinant()	—
M.height()	global height
M.inverse()	—
M.LLL([args])	LLL reduced lattice
M.minors(k)	dets of $k \times k$ minors
M.rank()	—

Polynomial Rings

R.<a,b>=PolynomialRing(K, 2)	poly ring over K
R.<a>=PolynomialRing(K)	univar poly ring
R.<a>=PolynomialRing(K, 1)	multivar poly ring
R.gen(k)	kth variable
R.gens()	all variables
R.hom(im_gens, S)	Hom(R, S)
R.ideal(polys)	—
I.dimension()	krull dim of R/I
I.elimination_ideal(vars)	—
I.gens()	—
I.groebner_basis()	—
I.is_prime()	—
I.is_maximal()	—
I.is_principal()	—
I.is_one()	—
I.primary_decomposition()	—
I.radical()	—
I.ring()	R
I.variety()	rat pts of dim 0
I.vector_space_dimension()	of R/I
F.monomial_coefficient(mon)	base ring element
F.polynomial(x)	make univariate
F.subs(dict)	substitution
F(tuple)	substitution
F.coefficient(mon)	poly element
F.coefficients()	—
list(F)	list of (coeff, mon)
F[list]	coeff of mon with exp list
F.dict()	dict of mon:coef via exp
F.lift(I)	coeff of gens of I to get F

Algebraic Geometry

S.Chow_form()	associated Chow form
S.coordinate_ring()	—
S.defining_ideal()	—
S.defining_polynomials()	—
S.degree()	from lc of hil poly
S.dimension()	relative dimension
S.intersection(T)	—
S.intersection_multiplicity(T, Q)	Serre's Tor
S.irreducible_components()	—
S.is_smooth()	—
S.Jacobian()	Jacobian ideal
S.projective_closure([P])	—
S.rational_points([bd])	—
S.subscheme(ideal)	—
S.veronese_embedding(d)	—
S.weil_restriction()	—
S*T	$S \times T$
S**n	$S \times \dots \times S$
I.radical()	radical ideal
PP.components()	—
PP.dimension_components()	list of dims
PP.segre_embedding([codomain])	—
C.arithmetic_genus()	—
C.genus()	—
C.is_complete_intersection()	—
C.is_ordinary_singularity(Q)	—
C.is_transverse(D, Q)	—
C.tangents(Q)	—

Copyright © 2017 B. Hutz, J. Silverman v3.0. Permission is granted for noncommercial distribution provided the copyright notice and this permission notice are preserved on all copies. Thanks to ICERM for hosting us while version 1.0 was written.