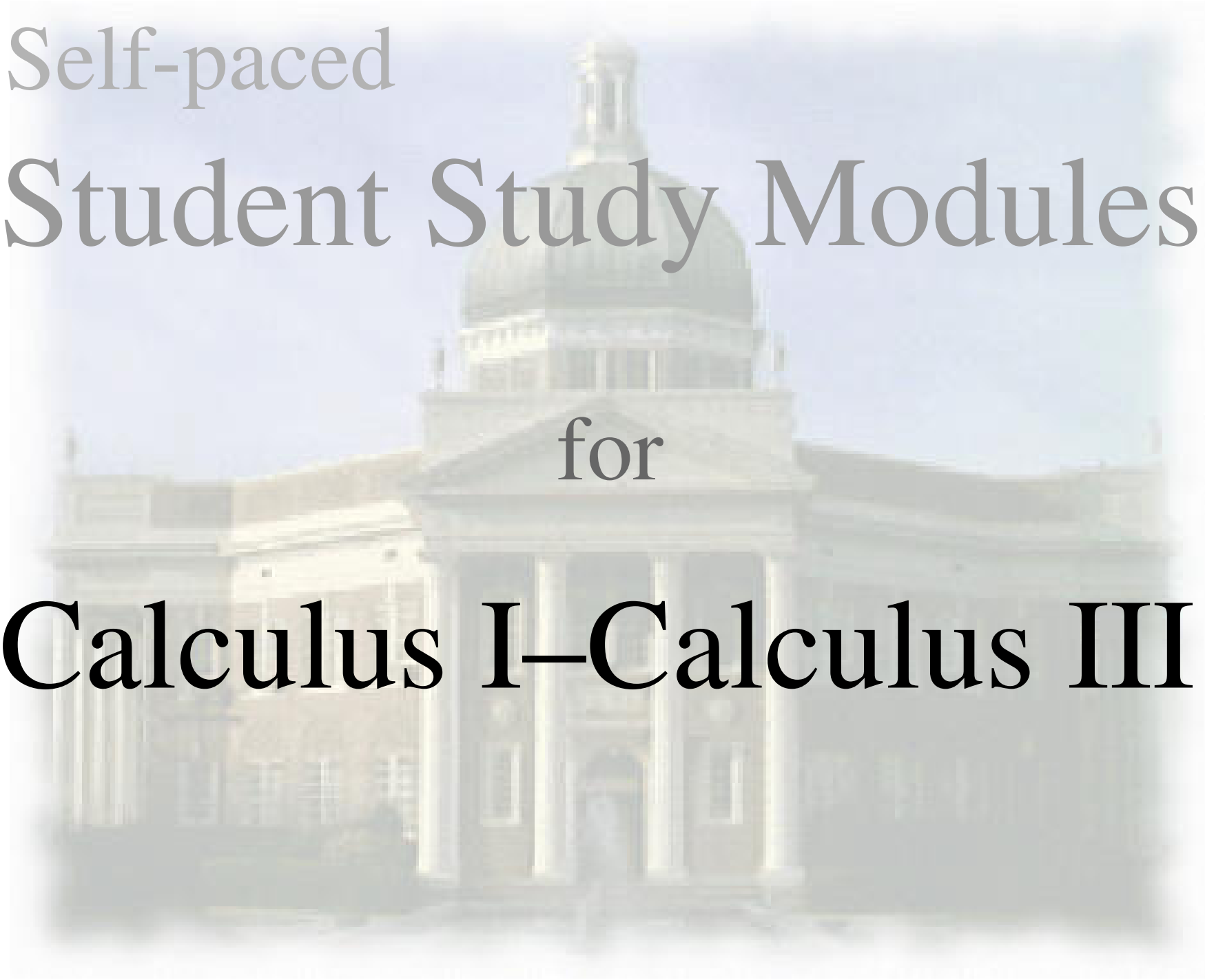


Self-paced

Student Study Modules

for

Calculus I–Calculus III

Instructions

This tutorial session is color coded to assist you in finding information on the page. It is online, but it is important to take notes and to work some of the examples on paper.

- You can move forward through the pages ⟨**Next**⟩, backward ⟨**Prev**⟩, or view all the slides in this tutorial ⟨**Index**⟩.
- The ⟨**Back to Calc I**⟩ button returns you to the course home page.
- A full symbolic algebra package ⟨**Sage**⟩ is accessible online. You can download and install it on your own computer, without a web app, by visiting `www.sagemath.org`.
- An online calculus text ⟨**CalcText**⟩ provides a quick search of basic calculus topics.
- You can get help from Google Calculus ⟨**GoogleCalc**⟩.
- A monochrome copy of this module is suitable for printing ⟨**Print**⟩.

When all else fails, feel free to contact your instructor.

An Introduction to SAGE

These modules are aimed as a series of online, interactive lessons and labs to help you learn Calculus. An essential component of the modules is the use of the computer algebra package SAGE. SAGE is a state-of-the-art, free computer algebra system that allows you to do many different kinds of mathematics, from basic arithmetic to calculus and beyond. Similar commercial systems include Maple and Mathematica.

Some of the lessons advise you to use SAGE. In some cases you will use prebuilt applications we call SAGE applets; in other cases you will start a blank worksheet and go from there. You are welcome to use it on your own to try to answer your questions or explore your ideas. The reason we include SAGE is because *we want you to experiment, even play with the mathematics.*

Because SAGE is essential for this aspect of the course, this lesson is designed to serve as an introduction to some basic constructions of SAGE.

Getting started

You have several options for obtaining and running SAGE. All of them are free.

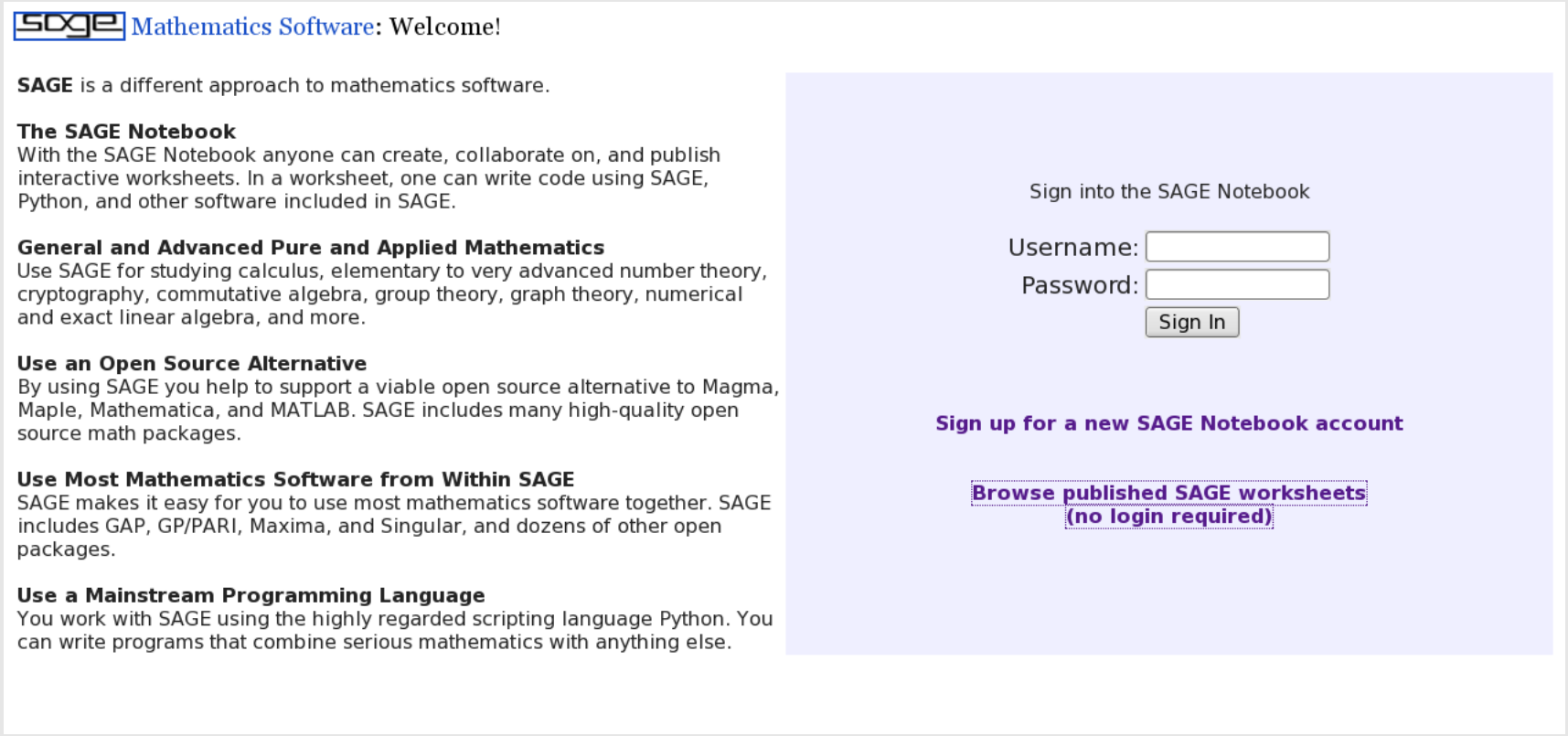
- Visit the department's SAGE server at `https://sage.st.usm.edu:8000/` There you can login as guest (no password), or create your own username. This can be slow if there is a great deal of demand for the server, and the server might be down sometimes.
- Obtain a copy of SAGE on a CD-ROM from the math department, and run it on your own computer.
- Download SAGE from `http://www.sagemath.org/` and run it on your own computer.

You will sometimes need to use worksheets that the math department has prepared for this course. You should be able to find these at the same website where you find the modules, `http://www.math.usm.edu/sage/`

If you need help obtaining SAGE or the worksheets, your professor should be able to help you. If not, s/he will tell you whom you can contact.

Start SAGE

At this point we assume that you have obtained SAGE and have started it, either by navigating to the web address online or by double-clicking an icon on your computer. You may receive some security warnings; allow any certificates to pass. (The Firefox web browser calls this "adding an exception.") You should see a welcome screen:



(Colors and some minor details may vary.)

Log in to SAGE

SAGE's worksheet interface works best with the Firefox web browser, which you can download (for free!) at `http://www.mozilla.com/` It should also work with Internet Explorer, Safari, Opera, and any web browser.

Go ahead and sign up for a new SAGE Notebook account. Then log in to your account. You should see a screen like this:

Your first SAGE worksheet

In the upper-left hand region of the window, you will see a link for creating a new worksheet. Click on that, and you will see something like the following.

Pay particular attention to the two circled objects.

- The one on the left is the title of your worksheet. You can change it by clicking on it and typing a new name. Do that now; change the title to *My first SAGE worksheet!*
- The one on the right is a link to help. SAGE comes with a substantial amount of documentation, and you may want to explore it from time to time. However, we will try to include in this module everything you need to know to make SAGE work.

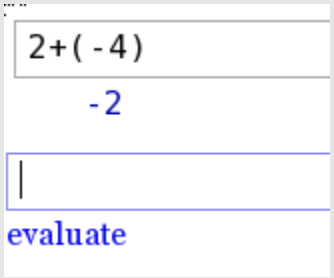
Cells

When you want SAGE to do something, you type instructions into a *cell*. A cell looks like a rectangle; we have written the word "CELL" in the screenshot below to show you where it is.

Click in the cell, and you will be ready to work with SAGE.

Basic Arithmetic

SAGE will perform basic arithmetic like any calculator. Type $2+(-4)$ into the cell. To make SAGE perform an instruction, hold the <SHIFT> key and press <RETURN> (or <ENTER>). You should see this:



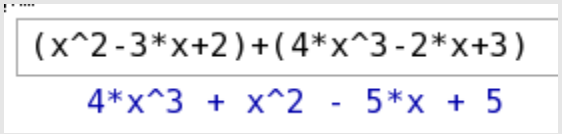
Personally, I find it highly reassuring that SAGE agrees with every other calculator on this point.

Symbolic calculator

SAGE is more than a calculator, however; it is a *Computer Algebra System*. Unlike ordinary calculators, SAGE can compute with polynomials, matrices, and esoteric mathematical objects that you've probably never heard of, such as Gröbner bases, modular forms, elliptic curves, finite fields, and lots more.

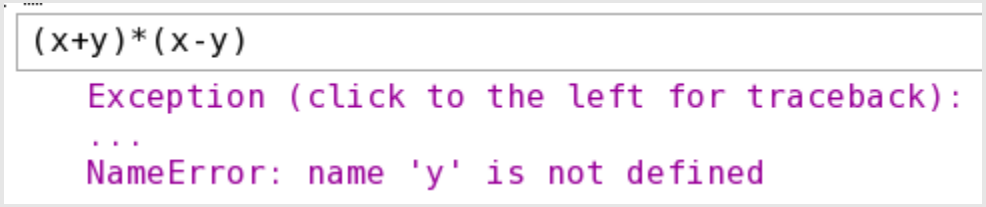
Let's add two polynomials. We'll shoot for something that you should be able to check yourself: $(x^2-3x+2)+(4x^3-2x+3)$. To type this in, use the caret (^) to show exponentiation, and a star (*) to show multiplication. Unlike written mathematics, you will need to include the star for every multiplication.

We get this:

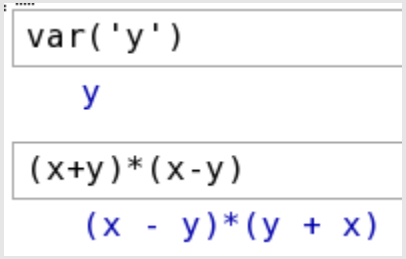


Declaring variables

SAGE recognizes x as a variable. If you want your functions to use variables other than x, you will have to tell this to SAGE. Otherwise you could get an error like the following.



To declare y as a variable to be used for functions, type `var('y')`. After that you can manipulate it to your heart's content.



You can declare more than one variable by typing, for example, `a, b, c = var('a b c')`.

Simplifying expressions

You may have noticed in the previous slide that SAGE did not expand the product $(x+y)(x-y)$, but returned it to us in the same form. If you want SAGE to expand the product, you can tell it to do so explicitly.

```
expand((x+y)*(x-y))
```
```
x^2 - y^2
```

Likewise, if you want it to factor a polynomial, it will do so if it can.

```
factor(x^4-4*x^3*y+6*x^2*y^2-4*x*y^3+y^4)
```
```
(x - y)^4
```

SAGE will also solve equations, but you have to use two equality signs instead of one:

```
solve(2*x^2+4*x==-2)
```
```
[x == -1]
```

SAGE has a large number of useful functions like these. We will introduce the functions that you need when necessary.

Multiple commands

You can enter multiple commands into a cell, separating each one by pressing <ENTER>. Do not hold the <SHIFT> key when you want to enter multiple commands into a cell.

```
(x+y) + (x-y)
(x+y) - (x-y)
```

```
2*y
```

Notice that SAGE only printed the result of the *last* command in the cell. To print the results of all the commands in a cell, surround each command with the `print` function.

```
print((x+y) + (x-y))
print((x+y) - (x-y))
```

```
2 x

2 y
```

Naming expressions

It can grow tiresome to retype the same expressions. Assigning names to expressions makes this less burdensome. You can make an assignment this way:

<name> = <expression>

To illustrate, create two polynomials f and g, then perform a series of operations on them. Notice that SAGE does not print the result of an assignment.

```
f = x + y
g = x - y
```

```
f + g
```

```
2*x
```

```
f - g
```

```
2*y
```

```
f*g
```

```
(x - y)*(y + x)
```

```
f/g
```

```
(y + x)/(x - y)
```

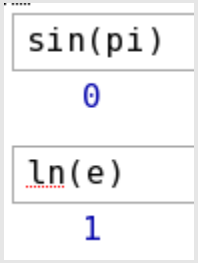
```
expand(f^2*g^3)
```

```
-y^5 + x*y^4 + 2*x^2*y^3 - 2*x^3*y^2 - x^4*y + x^5
```

Special names

SAGE reserves some names for mathematical expressions. The ones that matter for Calculus are

- `pi` for π and
- `e` for e.



Plotting graphs

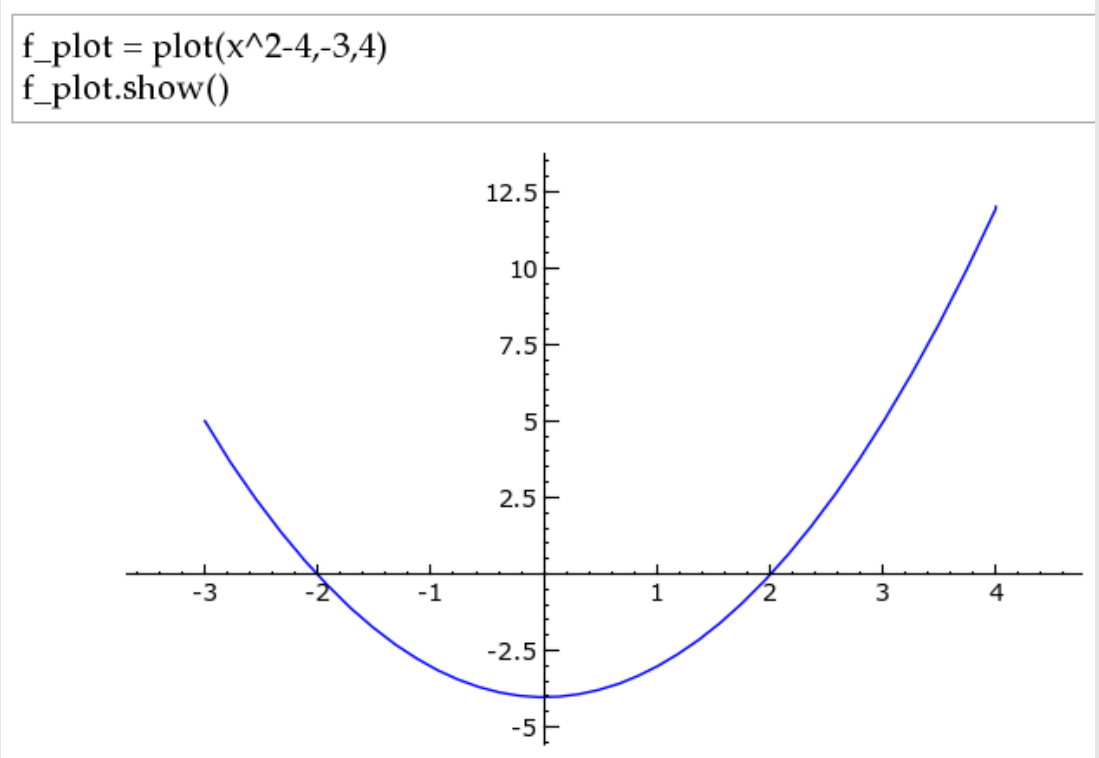
You can plot a graph using the commands `plot()` and `show()`. To plot x^2-4 on the interval $[-3,3]$, type

```
plot(x^2-4,-3,3)
```

You can also assign the plot to a name such as `f_plot` and then type

```
f_plot.show()
```

You would see something like this:

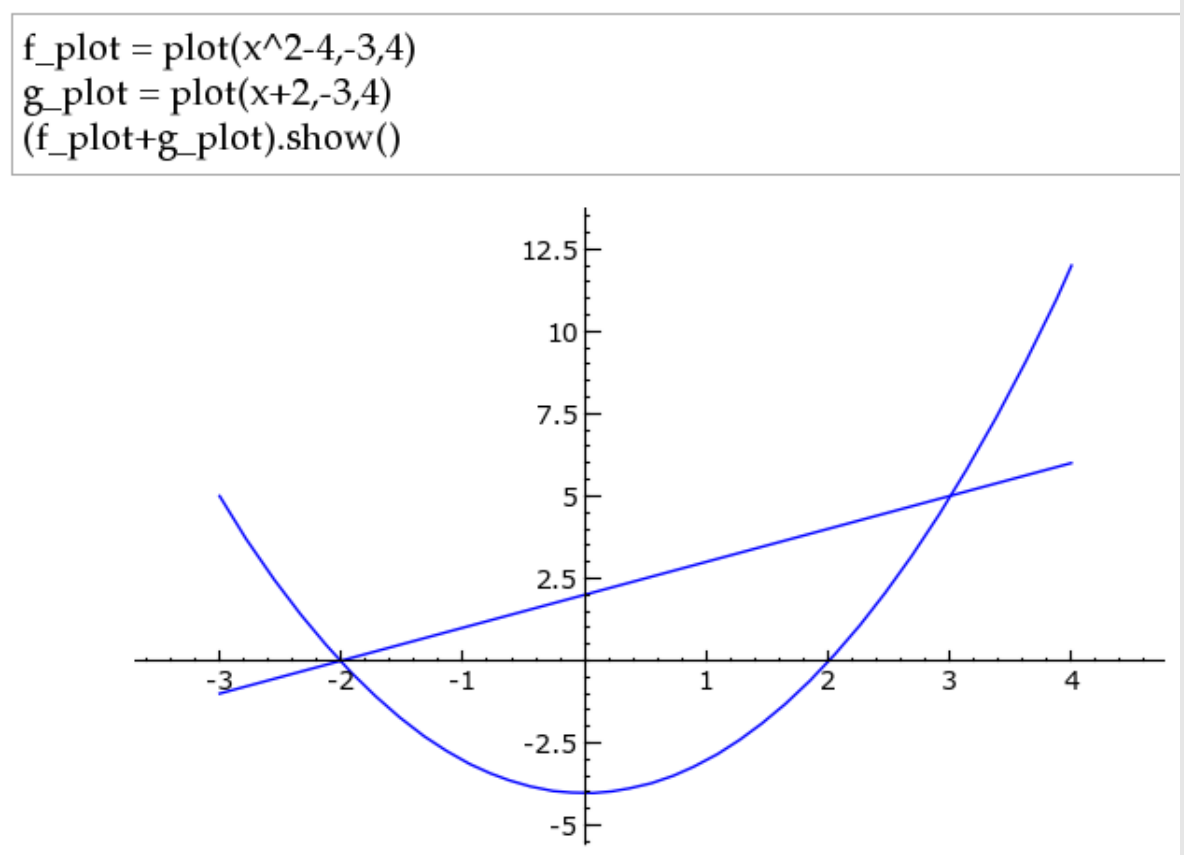


Combining the plots of graphs

You can also combine several plots into one. Assign the plots of x^2-4 and $x+2$ over the interval $[-3,3]$to the names `f_plot` and `g_plot`, then type

```
(f_plot + g_plot).show()
```

You will see something like this:

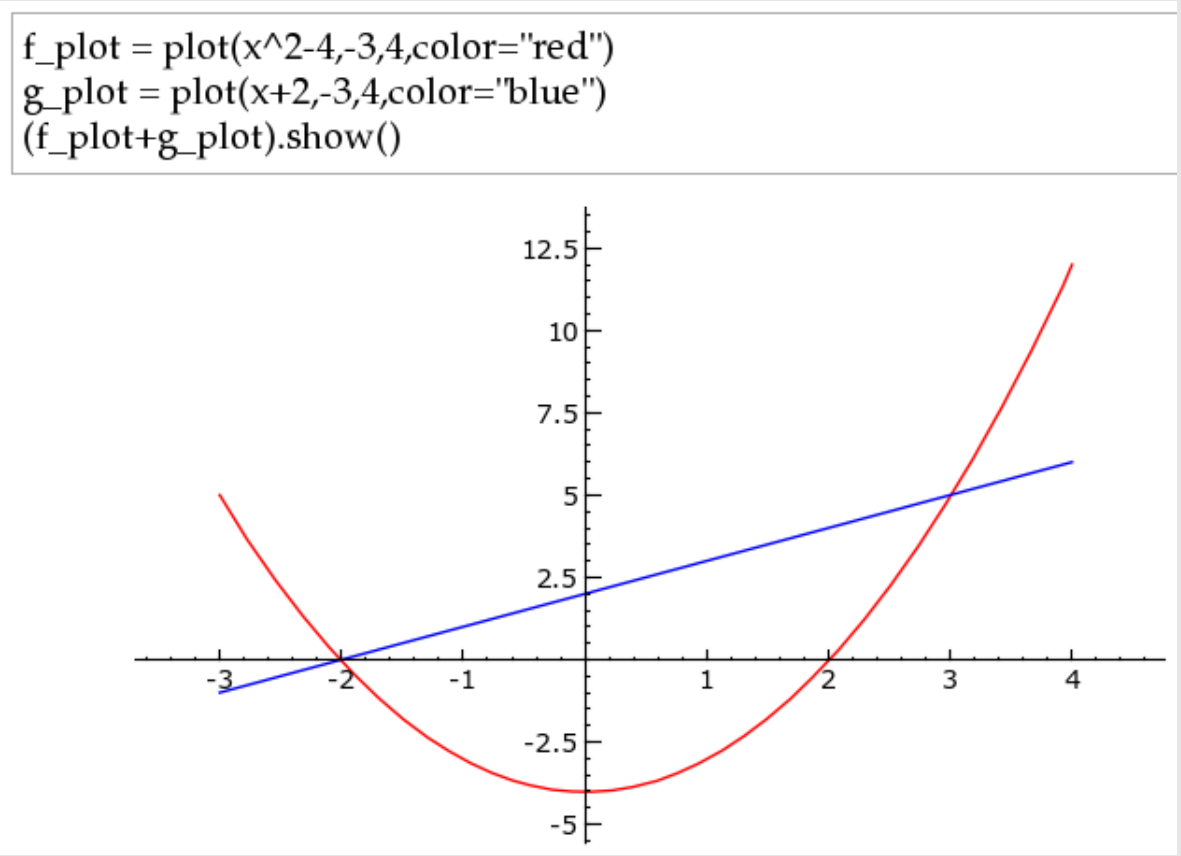


Colored plots

When combining plots, it can be hard to tell them apart if they are all the same color. SAGE has an option for changing the color of a plot. Type the following:

```
f_plot = plot(x^2-4,-3,4,color="red")
g_plot = plot(x+2,-3,4,color="blue")
(f_plot + g_plot).show()
```

You will see this:



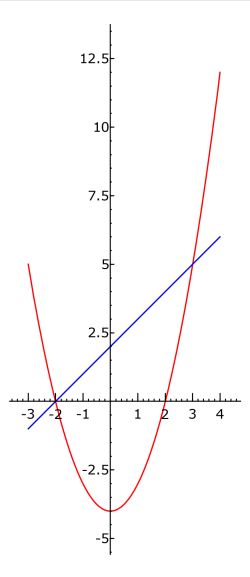
Adjusting the aspect ratio

You may have noticed that the x-axis is a little longer than the y-axis. The result is that the graphs are a little distorted; a circle, for example, would look like an ellipse.

You can fix this by adding an option in the `show()` command called `aspect_ratio`. If you specify `aspect_ratio=1`, then the graph will look square. For example,

```
(f_plot + g_plot).show(aspect_ratio=1)
```

generates the following graph.



Adding, removing cells

If you want to insert a cell between two others, hover the mouse slightly above an old cell. A line should appear:

If you click on that line, a new cell will appear.

To delete a cell, delete its contents, place the cursor inside the cell, then press `<Backspace>`

Values of names

SAGE remembers the value of a name based on the order it was executed, *not* the order it appears in the worksheet. If you insert a cell between two others and each of the three assigns a value to the name f, you could see something like this:

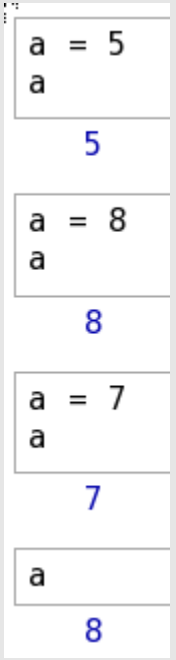


Figure 1: Why is a equal to 8 in the last cell?!?

Values of names

Why does *a* have the value 8 in the last cell? Because the second cell was executed *after* the third one. This is not something that we can show you in the slides, but you can try it yourself:

- type what is written in the first three cells of Fig. 1 and execute them (hold <SHIFT> and press <ENTER>);
- execute the second cell again (click within the cell, hold <SHIFT>, and press <ENTER>;
- *without executing the third cell again*, click within the fourth cell, type what you see in Fig. 1, and execute it.

If calculations aren't working the way the worksheet suggests, it is probably because something has been executed out of order. You can fix this by executing the cells again in the proper order.

Saving a worksheet

You can—and should—save your work frequently using the **<Save>** button in the upper right. If you want to work on a new worksheet, or just leave SAGE, you have several options:

- the **<Save and quit>** button;
- the **<Discard and quit>** button;
- the **<New worksheet>** option in the **<File...>** drop-down, and
- the **<Home>** link,

For now choose the **<Save and quit>** option. It will return you to the home screen, and you will find your worksheet listed!

Loading a worksheet

For many of the modules you will have to load a SAGE worksheet that contains a SAGE applet. We will describe three ways that you can do this.

Loading a worksheet

For many of the modules you will have to load a SAGE worksheet that contains a SAGE applet. We will describe three ways that you can do this.

First, you can visit the SAGE homepage and download the module.

- The SAGE homepage is `http://www.math.usm.edu/sage/`.
- Find the link to module worksheets, and click on it.
- Find the link to the worksheet that you want, and click on it.
- On the SAGE homepage, you will see a link named **<Upload>** in the upper left, next to the **<New worksheet>** link.
- Click on the **<Browse...>** button, and browse to the location where you saved the worksheet on your computer. Select your worksheet.
- Press the **<Upload Worksheet>** button. You will now return to the list of worksheets, and the worksheet that you uploaded will be available.

Loading a worksheet

For many of the modules you will have to load a SAGE worksheet that contains a SAGE applet. We will describe three ways that you can do this.

Second, you can load it from a disk, such as a CD-ROM that your instructor gave you.

- On the SAGE homepage, you will see a link named **<Upload>** in the upper left, next to the **<New worksheet>** link.
- Click on the **<Browse...>** button, and browse to the location where the worksheet lies on the CD-ROM. (Look for a directory called `Worksheets`.)
- Press the **<Upload Worksheet>** button. You will now return to the list of worksheets, and the worksheet that you uploaded will be available.

Loading a worksheet

For many of the modules you will have to load a SAGE worksheet that contains a SAGE applet. We will describe three ways that you can do this.

Third, you can load it directly from another website. In this case your instructor may have created his own worksheet that he wants you to use.

- On the SAGE homepage, you will see a link named **<Upload>** in the upper left, next to the **<New worksheet>** link.
- There is a box labeled, *Or enter the url of a worksheet file on the web:*. In this box, type the address that your instructor gave you.
- Press the **<Upload Worksheet>** button. You will now return to the list of worksheets, and the worksheet that you uploaded will be available.

Programming

One of the strengths of the SAGE computer algebra system is that it relies on a computer programming language called Python. You don't have to know any Python in order to complete the worksheets for this course. However, you will learn a little bit about Python programming indirectly, and if you do know how to program in Python then you can do many powerful things in SAGE.

End of Module

Please review your work, select another module, or select an option from the top menu.

You may also obtain a black and white condensed version of this tutorial by clicking the ⟨**Print**⟩ icon, and then saving or printing the pdf file.

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