Self-paced

Student Study Modules

for

Calculus I–Calculus III

Instructions

This tutorial session is color coded to assist you in finding information on the page. It is online, but it is important to take notes and to work some of the examples on paper.

- You can move forward through the pages ⟨**Next**⟩, backward ⟨**Prev**⟩, or view all the slides in this tutorial ⟨**Index**⟩.
- The ⟨**Back to Calc I**⟩ button returns you to the course home page.
- A full symbolic algebra package ⟨**Sage**⟩ is accessible online. You can download and install it on your own computer, without a web app, by visiting `www.sagemath.org`.
- An online calculus text ⟨**CalcText**⟩ provides a quick search of basic calculus topics.
- You can get help from Google Calculus ⟨**GoogleCalc**⟩.
- A monochrome copy of this module is suitable for printing ⟨**Print**⟩.

When all else fails, feel free to contact your instructor.

Overview

Until this point you have used the derivative as the instantaneous rate of change of the function f at a *specific* point $x=a$. Now we let a be a variable, and try to compute *a general formula* for the derivative of f at a. This function is commonly called the derivative of f. (This is different from "the derivative of f *at the point* $x=a$.")

Geometrically, the instantaneous rate of change is the slope of a line tangent to f at $x=a$. In this lab, you will explore how we can use the tangent lines to deduce a formula for the derivative from its values at various points.

A Sample problem

Question: If $f(x)=\sin x$, what is $f'(x)$ *as a function?*
That is, what formula gives us $f(x)$ for any value of x?

SAGE worksheets

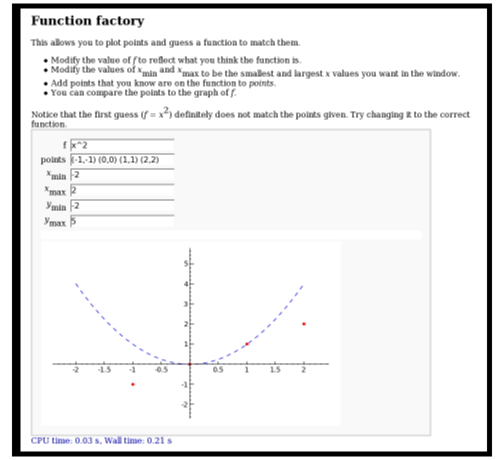
In this lab you will need the following SAGE worksheets:

- Function Factory
- Tangent Line Guesser

Open each one in separate windows or tabs of your browser.

Setup Function Factory

First we will set up the Function Factory SAGElet. When you first start it, you should see something like this:



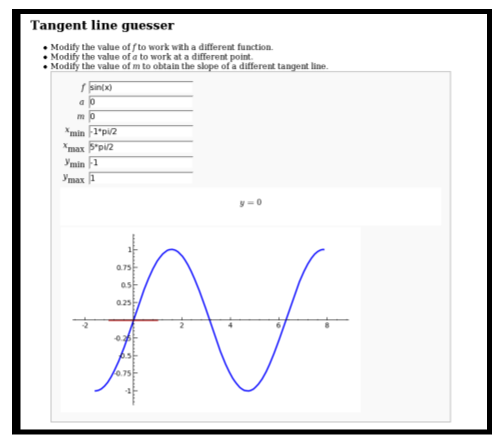
Setup Function Factory

Change the values of the input boxes to match the following:

- delete everything in *points*, leaving it empty;
- $x_{\text{min}} \rightarrow -pi/2$;
- $x_{\text{max}} \rightarrow 5*pi/2$.

Setup Tangent Line Guesser

Now turn to the Tangent Line Guesser SAGElet. You should see something like this:



Things are already set up for this exercise.

Estimating the derivative

We can estimate the values of the derivative of f using the Tangent Line Guesser. For any x value a, you can change the value of m, the slope of the red line.

Estimating the derivative

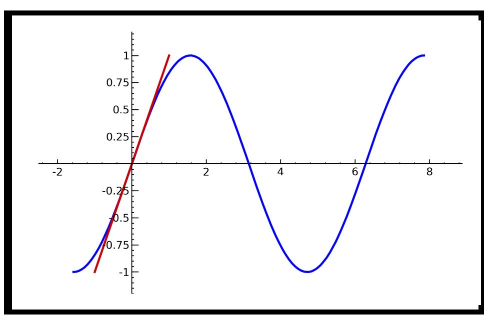
We can estimate the values of the derivative of f using the Tangent Line Guesser. For any x value a, you can change the value of m, the slope of the red line.

Experiment with different values of m until the red line seems to be tangent to the curve. For $a=0$, try $m=2$, $m=1$, $m=0.5$, $m=0$, and $m=-1$. Which value of m gives the correct line?

Estimating the derivative

We can estimate the values of the derivative of f using the Tangent Line Guesser. For any x value a, you can change the value of m, the slope of the red line.

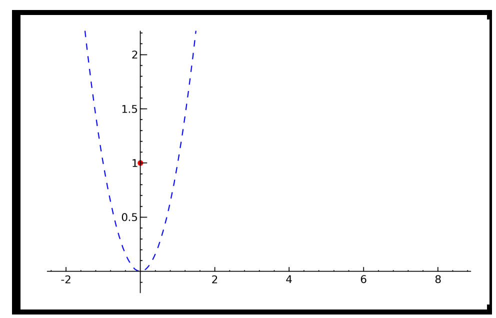
Experiment with different values of m until the red line seems to be tangent to the curve. For $a=0$, try $m=2$, $m=1$, $m=0.5$, $m=0$, and $m=-1$. Which value of m gives the correct line?



The value $m=1$ appears to create a tangent line. Now turn to the Function Factory.

Points in derivative

In the Function Factory SAGElet, we enter points that belong to the derivative, and see if they match the function specified by f. We decided in the Tangent Line Guesser SAGElet that $f'(0)\approx 1$. Type (0,1) in the input box next to *points*. Press the tab or enter key to see an update. You should see something like this:



Right away you can see that x^2 is *not* the correct derivative for $\sin x$, because the point (0,1) does not lie on the curve. However, we won't change f yet.

More points!

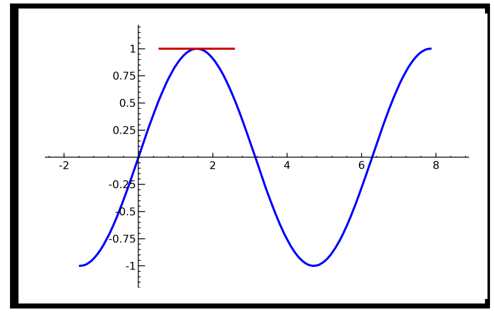
Return to the Tangent Line Guesser SAGElet. This time, try different values for a. At each value of a, try to find a value of m that gives the correct tangent line. For example:

Assignment: Derivatives by experimentation 7

More points!

Return to the Tangent Line Guesser SAGElet. This time, try different values for a. At each value of a, try to find a value of m that gives the correct tangent line. For example:

When we try $a = \pi/2$, we find $m \approx 0$.

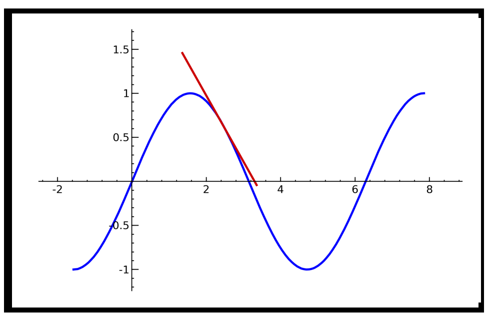


Assignment: Derivatives by experimentation 7

More points!

Return to the Tangent Line Guesser SAGElet. This time, try different values for a. At each value of a, try to find a value of m that gives the correct tangent line. For example:

When we try $a = 3\pi/4$, we find $m \approx -0.75$.

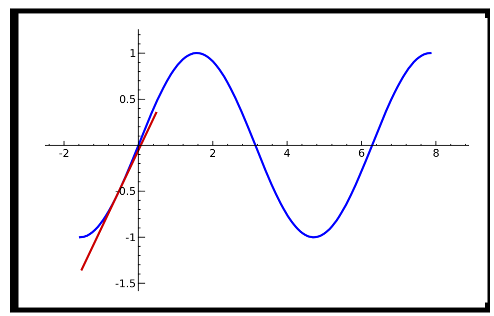


Assignment: Derivatives by experimentation 7

More points!

Return to the Tangent Line Guesser SAGElet. This time, try different values for a. At each value of a, try to find a value of m that gives the correct tangent line. For example:

When we try $a = -\pi/6$, we find $m \approx 0.85$.

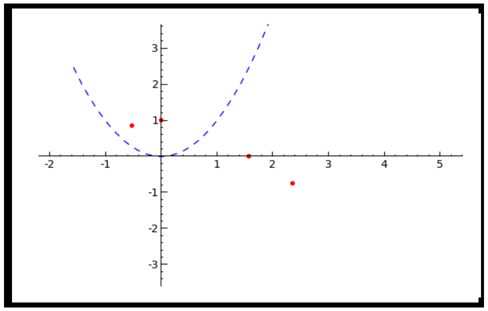


Back to Function Factory

Add the points

$$\left(-\frac{\pi}{6}, 0.85\right), (\pi/2, 0), \left(\frac{3\pi}{4}, -0.75\right)$$

into Function Factory and you get the following image.



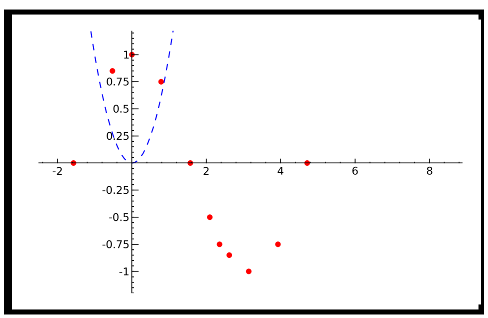
At this point we can see that x^2 is *definitely* wrong. But we still don't know what the correct function is!

Take some more points

The only thing we can do now is try some more values of a. Choose values that lie between and outside the values we have used so far. You might want to try $a = \pi/4$, for example, or $a = 5\pi/4$, or $a = 3\pi/2$, or more! For each value of a, find the corresponding value of m. When you think you have enough points, add them to Function Factory.

Things fall into place

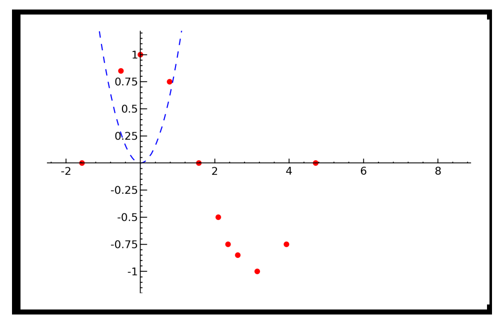
Here is an image reflecting those choices, as well as some others:



You should be able to guess the correct value of f at this point...

Things fall into place

Here is an image reflecting those choices, as well as some others:



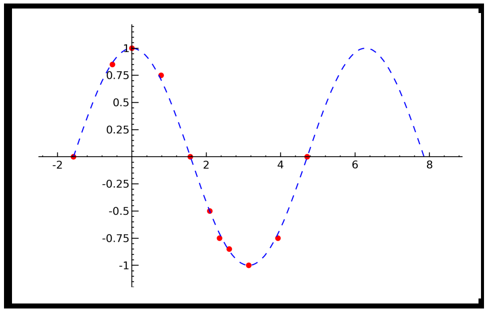
You should be able to guess the correct value of f at this point... the correct answer is

$$f = \cos x.$$

Change the value of f in the Function Factory to $\cos x$.

Things fall into place

It looks as if $\cos x$ matches the points perfectly.



Try more!

Repeat this exercise with several different functions. (Your professor may assign a particular function to you.) Enter each function into the Tangent Line Guesser SAGElet, then determine approximate values of the derivative. Enter the appropriate points into the Function Factory SAGElet, repeating until you have found enough points to guess a formula for the derivative.

e^x	$\ln(x)$
x^2	$\sqrt{x}$
$\cos(x)$	$\sin(2x)$
$\tan(x)$	$\sin(x+\pi/2)$

Conclusion

In class you will study and learn algebraic methods that should explain why you obtain the result you obtained here. Feel free to experiment with other functions.

End of Module

Please review your work, select another module, or select an option from the top menu.

You may also obtain a black and white condensed version of this tutorial by clicking the ⟨**Print**⟩ icon, and then saving or printing the pdf file.

Department of Mathematics at
The University of
Southern Mississippi