Drinfeld cusp forms and harmonic cocycles

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Outline

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Drinfeld modular forms
The Bruhat-Tits tree
The Drinfeld upper half plane
Drinfeld modular forms
Harmonic cocycles
Definition
Remarks
The residue map
Fourier coefficients
Towards an algorithm
"Effective version of automatic cuspidality"
$\Gamma\backslash\mathcal{T}$ as a covering of a half line
References

Basic notation

$\mathbb{F}_q$ the field of $q = p^n$ elements
K_∞ a local field with residue field $\mathbb{F}_q$
$\mathcal{O}_\infty$ the ring of integers of K_∞
π_∞ a uniformizer of K_∞

The Bruhat-Tits tree

K_∞, $\mathcal{O}_\infty$, π_∞, $\mathbb{F}_q$

Definition (Bruhat-Tits tree)

$\mathcal{T}$:= the simplicial complex of dimension 1 with

- set of vertices $\mathrm{Vert}(\mathcal{T}) :=$ homothety classes $[\mathrm{L}]$ of rank 2 $\mathcal{O}_\infty$-lattices $\mathrm{L} \subset \mathrm{K}_\infty^2$
- set of edges $\mathrm{Edge}(\mathcal{T}) :=$ pairs $([\mathrm{L}], [\mathrm{L}'])$ such that $\pi_\infty \mathrm{L} \subsetneq \mathrm{L}' \subsetneq \mathrm{L}$.

$|\mathcal{T}|$ the geometric realization of $\mathcal{T}$

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Lemma

$\mathcal{T}$ is a $q+1$-regular tree.

Definition

Particular vertices $\Lambda_i := [\mathcal{O}_\infty \oplus \pi_\infty^i \mathcal{O}_\infty]$, $i \in \mathbb{Z}$.
standard vertex Λ_0, standard edge $e_0 := (\Lambda_0, \Lambda_1)$

Group action on $\mathcal{T}$

$$K_\infty,\ \mathcal{O}_\infty,\ \pi_\infty,\ \mathbb{F}_q$$

Consider elements of K_∞^2 as **column vectors**
$\rightsquigarrow$ have natural left action of $GL_2(K_\infty)$ on K_∞^2.

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Definition ($GL_2(K_\infty)$-action on $\mathcal{T}$)

$$GL_2(K_\infty) \times \mathcal{T} \to \mathcal{T} : (\gamma, [L]) \mapsto [\gamma L]$$

Set $\Gamma_\infty := \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in GL_2(\mathcal{O}_\infty) \mid c \in \pi_\infty \mathcal{O}_\infty \right\}$.

Lemma

$GL_2(K_\infty)$ acts transitively on $Vert(\mathcal{T})$ and $Edge(\mathcal{T})$.

$Vert(\mathcal{T}) = GL_2(K_\infty)/GL_2(\mathcal{O}_\infty)K_\infty^*$,
$Edge(\mathcal{T}) = GL_2(K_\infty)/\Gamma_\infty K_\infty^*$.

Drinfeld's upper half plane Ω

$\mathbb{C}_\infty := \widehat{K_\infty^{alg}}$

Definition (Drinfeld's upper half plane)

$\Omega := \mathbb{P}^1(\mathbb{C}_\infty) \smallsetminus \mathbb{P}^1(K_\infty)$

Definition ($GL_2(K_\infty)$-action on Ω)

$GL_2(K_\infty) \times \Omega \to \Omega : (\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, z) \mapsto \gamma z = \frac{az+b}{cz+d}$

(rigid) analysis on Ω

$$K_\infty,\ \mathcal{O}_\infty,\ \pi_\infty,\ \mathbb{F}_q,\ \mathbb{C}_\infty,\ \Omega$$

Proposition (reduction map)

$\exists$ a (natural) $GL_2(K_\infty)$-equivariant map

$$\rho\colon \Omega \to |\mathcal{T}| \text{ such that}$$

$$\rho^{-1}(|e_0| \smallsetminus \{\Lambda_0, \Lambda_1\}) = \{z \in \mathbb{C}_\infty \mid 1 < |z| < q\}$$

$$\rho^{-1}(\Lambda_0) = \{z \in \mathbb{C}_\infty \mid |z| = 1\} \smallsetminus (q\text{-}1 \text{ open discs of radius } 1)!$$

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On these charts use Laurent series type expansions to define **(rigid) analytic functions on** Ω.

Drinfeld modular forms

From now on (for simplicity):

$$A = \mathbb{F}_q[T] \subset K = \mathbb{F}_q(T) \subset K_\infty = \mathbb{F}_q((\frac{1}{T})),\ \pi_\infty := \frac{1}{T}$$

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Fix $\Gamma \subset GL_2(A)$ a congruence subgroup:

Definition (Goss)

A **Drinfeld modular form of weight** k**, type** ℓ **for** Γ is a rigid analytic function $f : \Omega \to \mathbb{C}_\infty$ such that

(a) $f(\gamma z) = \det \gamma^{-\ell} (cz + d)^k f(z)$ for all $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$.

(b) Laurent series expansion of f at all cusps has vanishing principal part.

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Examples: The maps
(rank 2 Drinfeld module φ) $\mapsto$ the i-th coefficient of φ_a.
(rank 2 Drinfeld module φ) $\mapsto$ its discriminant.

Results

Define cusp forms (as usual).
Define Hecke operators for primes $0 \neq \mathfrak{p} \subset \mathbb{F}_q[T]$
Have no Petersson inner product for char. p forms.

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Let f be a Hecke eigenform with eigenvalues $a_{\mathfrak{p}}(f)$.

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$K_f := K(\{a_{\mathfrak{p}}(f)\}_{\mathfrak{p}})$ is finite over K.

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Theorem (B.)

(1) There is a strictly compatible system

$$\left(\rho_{f,\lambda}\colon Gal(\overline{K}/K) \to GL_1(\widehat{K_f}^{\lambda})\right)_{\lambda \text{ finite}}$$

such that $\rho_{f,\lambda}(Frob_{\mathfrak{p}}) = a_{\mathfrak{p}}(f)$ for almost all p.

(2) The sequence $(a_{\mathfrak{p}}(f))_{\mathfrak{p}}$ is given by a Hecke character!

Questions

General multiplicity one is wrong!

Does it hold in weight 2 and for $\Gamma_0(\mathfrak{p})$ with $\mathfrak{p}$ prime?

$\rightsquigarrow$ Possible implications for uniform boundedness of torsion points of Drinfeld modules of rank 2 over K. (C. Armana)

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Hecke characters and Galois reps. for DMF's:

Can have $\rho_{f,\lambda}(Frob_{\mathfrak{p}}) \neq a_{\mathfrak{p}}(f)$ for $\mathfrak{p} \not| N\mathfrak{p}_\lambda$ because of possible non-ordinary reduction of $X_0(N\mathfrak{p}_\lambda)$ at $\mathfrak{p}$

Determine such $\mathfrak{p}$ for $X_0(N)$; do they obey some patterns??

Compute ∞-types of the associated Hecke characters.

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Determine such $\mathfrak{p}$ for $X_0(N)$; do they obey some patterns??

Compute ∞-types of the associated Hecke characters.

Recover eigenforms from eigenvalues when possible?

Harmonic cocycles

$\mathcal{T}$, $Edge(\mathcal{T})$, Γ

"Drinfeld modular forms via local systems on trees":

Let M be a $K[GL_2(A)]$-module with $\dim_K(M)$ finite.

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Definition

$C_{har}(\Gamma, M)$:= the K-vector space of M-**valued** Γ-**invariant harmonic cocycles** := the set of maps

$$c\colon Edge(\mathcal{T}) \to M : e \mapsto c(e),$$

such that:

1. For all edges e one has $c(-e) = -c(e)$.
2. For all vertices v one has $\sum_{e\to v} c(e) = 0$, where the sum is over all edges e ending at v.
3. For all $\gamma \in \Gamma$ and $e \in Edge(\mathcal{T})$ one has $c(\gamma e) = \gamma c(e)$.

Remarks

$\mathcal{T}$, $Edge(\mathcal{T})$, Γ

(1)

Proposition (automatic cuspidality; Teitelbaum)

If M is p-power torsion then:
$\exists Z \subset \Gamma\backslash\mathcal{T}$ finite, s.t. all $c \in C_{har}(\Gamma, M)$ vanish outside Z.

$\rightsquigarrow$ space of harmonic cocycles is computable!

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(2) $C_{har}(\Gamma, \mathbb{Z}) \cong$ automorphic forms for Γ.

Proposition (Gekeler)

$C_{har\,!}(\Gamma, \mathbb{Z}) \otimes_{\mathbb{Z}} \mathbb{F}_p \subset C_{har}(\Gamma, \mathbb{F}_p)$ describes double cusp forms inside weight 2 cusp forms.

The residue map

$\mathcal{T}$, $Edge(\mathcal{T})$, Γ, $C_{har}(\Gamma, M)$

Recall:

A Drinfeld modular form f is a rigid analytic function on Ω.

Ω is a tubular neighborhood of $\mathcal{T}$ via ρ.

ρ^{-1} of the inner part of an edge e is an annulus $A(e)$.

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Define: For f of weight 2

$$\mathrm{Res}_2 : Edge(\mathcal{T}) \to \mathbb{C}_\infty : e \mapsto \mathrm{Res}_{A(e)}(fdz).$$

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Theorem (Teitelbaum)

$Res_2 : S_2^{Dr}(\Gamma, \mathbb{C}_\infty) \longrightarrow C_{har}(\Gamma, K) \otimes_K \mathbb{C}_\infty$ is an isomorphism of $\mathbb{C}_\infty$-vector space

An analogous theorem holds in weight k with $M \approx Sym^{k-2}$.

On the proof: Suffices for $\Gamma = \Gamma(N)$ for $N \in A \smallsetminus \mathbb{F}_q$.

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Construct $\mu_2 \colon C_{har}(\Gamma, \mathbb{C}_\infty) \to \mathrm{Meas}(\mathbb{P}^1(K_\infty), \mathbb{C}_\infty)^\Gamma$.

Use $\mathbb{P}^1(K_\infty) = \text{boundary}(\mathcal{T})$ with
$Edge(\mathcal{T}) \to$ basis of open sets of $\mathbb{P}^1(K_\infty) : e \mapsto U(e)$.

Define $c \mapsto \mu_{2,c}$ with $\mu_{2,c}(U(e)) := c(e) \quad \forall e$.

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Integrate against "Poisson-kernel":

$$\mathrm{Meas}(\mathbb{P}^1(K_\infty), \mathbb{C}_\infty)^\Gamma \to S_2^{Dr}(\Gamma, \mathbb{C}_\infty)$$

$$\mu \mapsto f_\mu(z) = \int_{\mathbb{P}^1} \frac{1}{z - \zeta} d\mu(\zeta)$$

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Theorem: The following composite is the identity:

$$(\mu \mapsto \text{Poiss.Int.}) \circ (c \mapsto \mu_{2,c}) \circ (f \mapsto \mathrm{Res}_2(f))$$

Corollary Res_2 is **injective**.

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Surjectivity: Compute dim $S_2^{Dr}(\Gamma, \mathbb{C}_\infty)$ via Riemann-Roch;
Compute dim $C_{har}(\Gamma, \mathbb{C}_\infty)$ combinatorially. Get equality. □

On Fourier coefficients

$\mathbf{e}$ = Carlitz exponential, $\widetilde{\pi}$ = Carlitz period, $t(x) := \mathbf{e}^{-1}(\pi x)$ uniformizer near cusp ∞, $f = \sum_{i \geq 1} a_i t^i$ a form for $SL_2(A)$

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Corollary

$$a_i = \widetilde{\pi} \int_{\pi_\infty \mathcal{O}_\infty} t^{1-i}(x) d\mu_f(x)$$

$\rightsquigarrow$ Can (in principle) recover the Fourier coefficients of a Drinfeld modular form f from its associated measure μ_f (in any weight)

"Effective version of automatic cuspidality"

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Definition (Serre?)

A simplex $t \in Vert(\mathcal{T}) \cup Edge(\mathcal{T})$ is Γ-**stable** iff

$$Stab_\Gamma(t) = \{1\}.$$

Proposition (Serre?)

There are only finitely many Γ-stable orbits of simplices.

These orbits are effectively computable! (see Ralf's talk)

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Theorem (Teitelbaum)

Suppose Γ is p'-torsion free. Then:

- *Any Γ-invariant harmonic cocycle is determined by its values on the Γ-stable orbits of edges.*
- *Relations: Only those from Γ-stable vertices.*

$\Gamma\backslash\mathcal{T}$ as a covering of a half line

How to understand the quotient $\Gamma\backslash\mathcal{T}$ and the Γ-stable simplices?

Proposition

The half line on $\{\Lambda_i\}_{i\geq 0}$ represents $GL_2(\mathbb{F}_q[T])\backslash\mathcal{T}$.

There are no $GL_2(\mathbb{F}_q[T])$-stable simplices of $\mathcal{T}$.
The stabilizers of Λ_i, $i \geq 1$, are strictly increasing in i.

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For general Γ:

$$\Gamma\backslash\mathcal{T} \to GL_2(\mathbb{F}_q[T])\backslash\mathcal{T}$$

is a finite, highly ramified 'covering' of the above half line.

"Monotonicity of the stabilizers" is inherited by $\Gamma\backslash\mathcal{T}$
$\rightsquigarrow$ stable simplices only above Λ_i for i small (depends on Γ).

For details on algorithms and some repetition: See Ralf's talk.

References

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