## Chapter 6: More advanced exercises

You will find in this tutorial a collection of more advanced exercises. You might also want to have a look at the worksheets on "Collatz conjecture", "Dictionaries and graph theory" and "Strings and the Burrows-Wheeler transform".

## Exercise 6.1

How many strings of length $n$ containing only letters $a$ and $b$ but not containing aa are there? Compute the value for each $n=1,2, \ldots, 10$

Do you know this sequence?
Could you find the corresponding entry in the OEIS

## Exercise 6.2 (Goldbach conjecture)

Verify experimentally the following statement: «For each even integer $n \geq 4$, there exist two prime number $p \geq 2$ and $q \geq 2$ such that $n=p+q$ ».
Up to which value of $n$ are you able to verify this conjecture?

## Exercise 6.3

Does there exist two positive integer numbers $x$ and $y$ such that $x^{2}-61 y^{2}=1$ ?

## Exercise 6.4

Write a function that given positive integers $(p, q, n)$ return the number of solutions in positive integers $\left(a_{1}, a_{2}, \ldots, a_{n}\right)$ to the equation

$$
\frac{1}{a_{1}}+\frac{1}{a_{2}}+\ldots+\frac{1}{a_{n}}=\frac{p}{q}
$$

How many solutions are there for $(p, q, n)=(13,12,3)$ ?

## A sample of Euler problems

Solve the following Euler problems

- problem 26 (decimal expansions)
- problem 31 (coin sums)
- problem 45 (triangular, pentagonal and hexagonal numbers)
- problem 46 (odd composite number that can not be written as the sum of a prime and twice a square)
- problem 50 (sum of consecutive primes that are primes)

