Number Theory and Arithmetic Dynamics Project

Bianca Thompson
Claremont, CA
Oct. 22-25, 2017

In arithmetic dynamics, we are interested in iterating maps via composition and looking for points that live in cycles (periodic points), eventually live in cycles (preperiodic points) or points that iterate forever (wandering points).

For example, the map $f(x) = x^2 - 1$ has 1 maps to 0 maps to -1 maps to 0. So 0 and -1 live in a cycle and 1 eventually maps to a cycle. Whereas 2 maps to 3 maps to 8 maps to 63 and so on under iteration, hence 2 is a wandering point.

In a finite space (more specifically a finite field), all points are either periodic or (strictly) preperiodic. Here we are interested in the proportion of points that strictly live in a cycle in a given finite field.

For example, if we work modulo 5, we can look at our above example again. The point 0 maps to 4 maps to 0 and 3 maps to 3, hence they are periodic while 1 and 2 are (strictly) preperiodic. Thus 3/5 of the time we can expect to find a periodic point here. If we fix a map and allow our finite field to vary, there is no reason to expect a detectable pattern of periodic points, but there are special families of maps we can look at where we do expect a pattern.

The goal of this project is to study special families of maps, called Lattès maps, and use SAGE to calculate the proportions of periodic points in different finite fields. We want to find patterns in the different families of Lattès maps and describe the behavior of proportions of periodic points over varying finite fields. SAGE is built with the capability of finding Lattès maps and looking at them over varying finite fields, so we will be relying on its built in programs.