sage-flatsurf (SageDays 74)

Speaker: Pat Hooper (http://wphooper.com) (City College of New York and CUNY Graduate Center)

Date: June 2nd, 2016. (This file was last updated on June 10, 2016.)

Sage-flatsurf is a program to explore "flat surfaces" by Vincent Delecroix and Pat Hooper.

It is available as a sage package at https://github.com/videlec/sage-flatsurf (https://github.com/videlec/sage-flatsurf).

Polygonal Billiards:

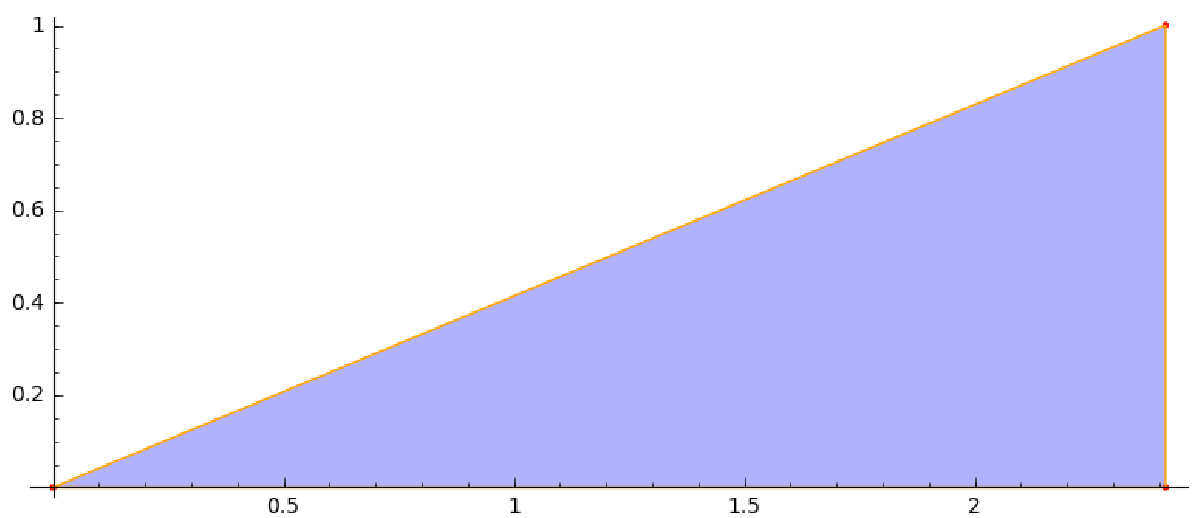
In [1]:
```
from flatsurf import *
from flatsurf.geometry.surface import Surface_list
```

The following builds a right triangle with one angle of pi/8 with a shorter leg of height 1.

In [2]:
```
p = polygons.right_triangle(1/8,leg1=1)
print(p)
p.plot()
```

```
Polygon: (0, 0), (a + 1, 0), (a + 1, 1)
```

Out[2]:



We use SAGE to represent the polygons so that the vertices have coordinates in a number field.

In [3]:
```
field = p.base_ring()
print(field)
a=field.gen()
print(str(a)+"="+str(a.n()))
print("p="+str(p))
```

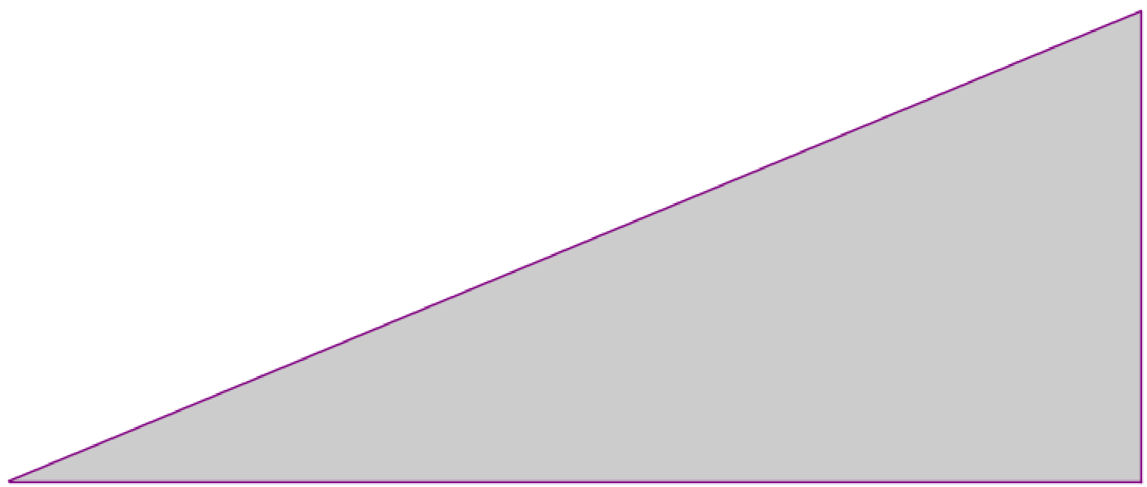
```
Number Field in a with defining polynomial y^2 - 2
a=1.41421356237309
p=Polygon: (0, 0), (a + 1, 0), (a + 1, 1)
```

We can make the polygon into a billiard table:

In [4]:
```
s = similarity_surfaces.billiard(p)
```

In [5]:
```
s.plot()
```

Out[5]:

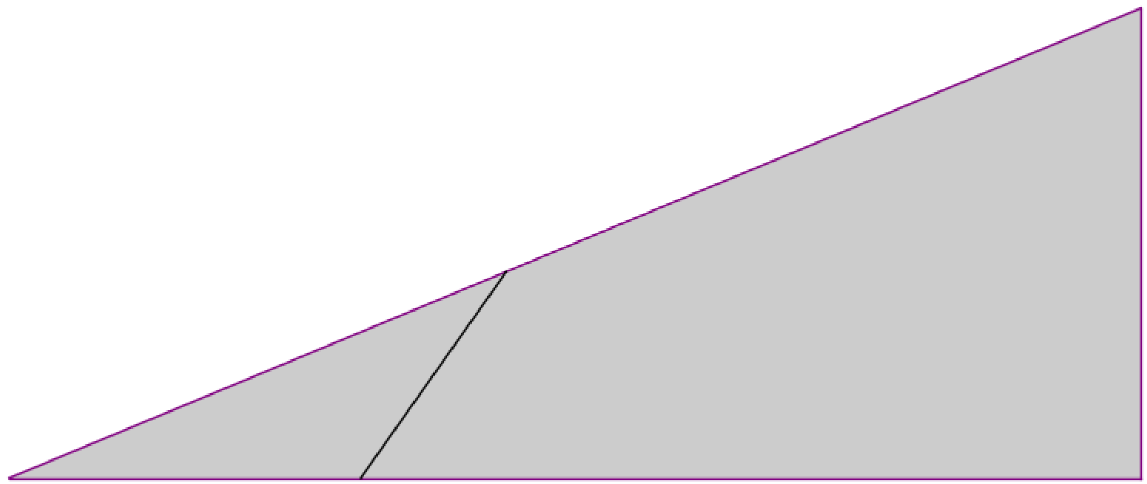


In [6]:
```
v = s.tangent_vector(0,(3/4,0),(1,a))
print(v)
```

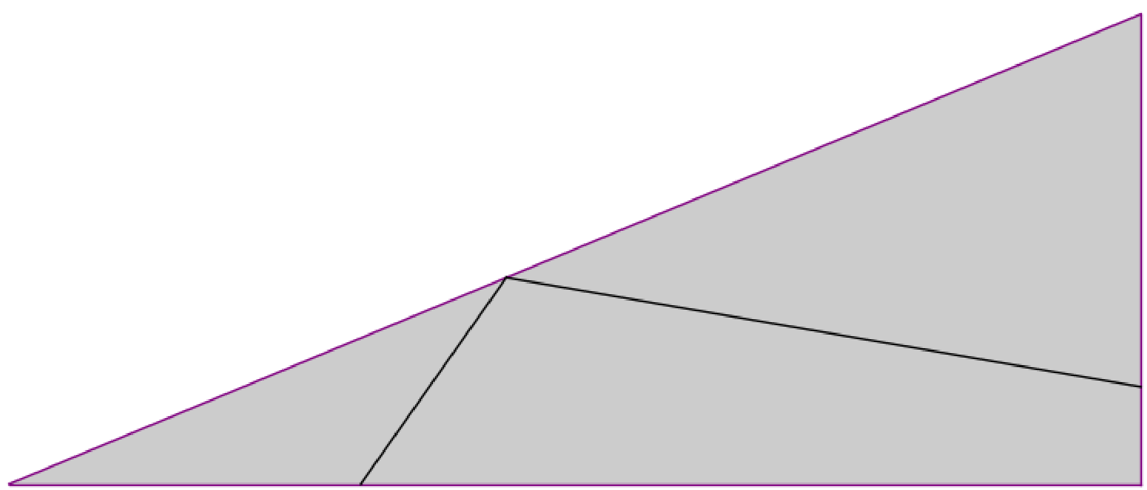
```
SimilaritySurfaceTangentVector in polygon 0 based at (3/4, 0) with vector
 (1, a)
```

In [7]:
```
traj = v.straight_line_trajectory()
```

In [8]:
```
s.plot()+traj.plot()
```
Out[8]:

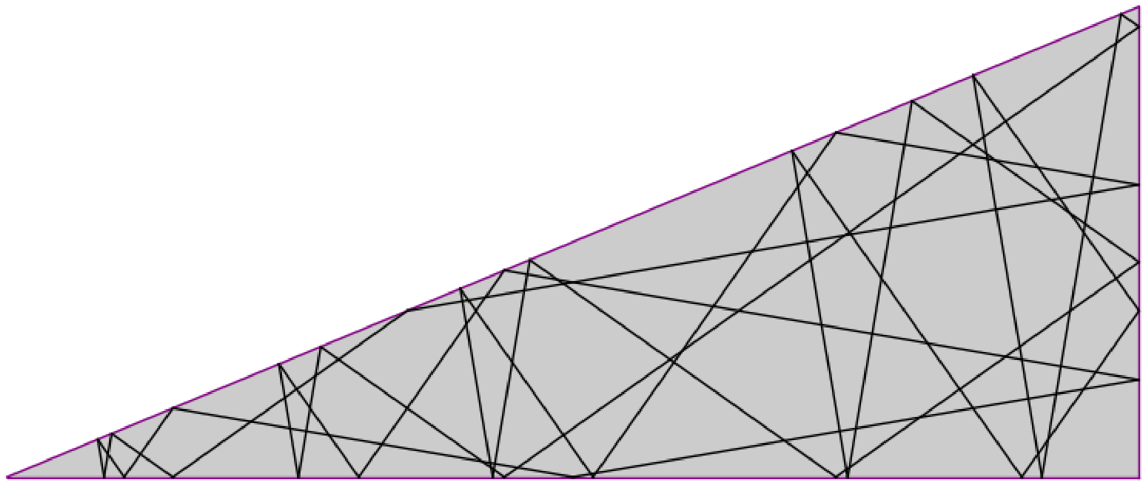


In [9]:
```
traj.flow(1)
```
In [10]:
```
s.plot()+traj.plot()
```
Out[10]:



In [11]:
```
traj.flow(100)
s.plot()+traj.plot()
```

Out[11]:



In [12]:
```
traj.is_closed()
```

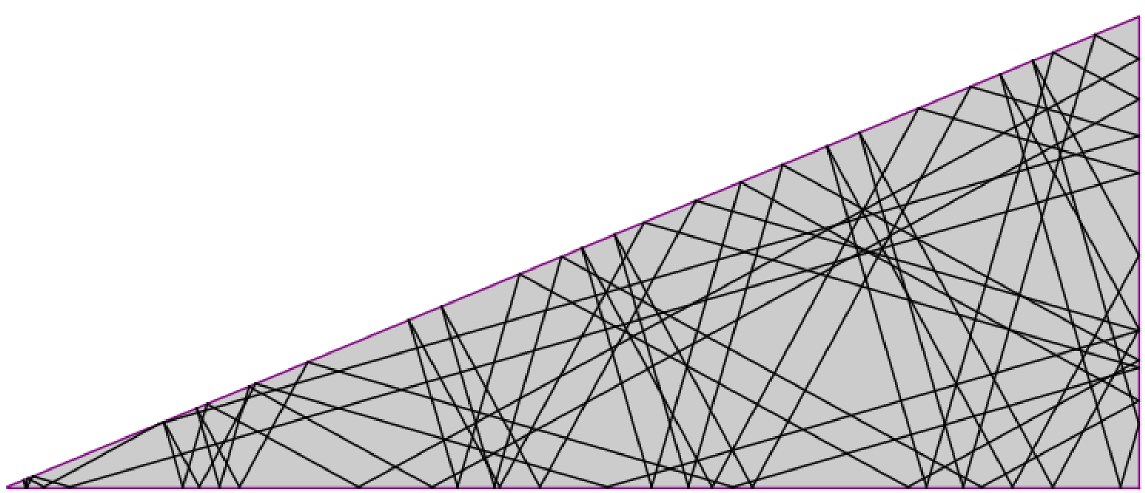
Out[12]: True

In [13]:
```
v = s.tangent_vector(0,(3/4,0),(3+a,1+a))
print(v)
traj = v.straight_line_trajectory()
traj.flow(1000)
```

```
SimilaritySurfaceTangentVector in polygon 0 based at (3/4, 0) with vector
 (a + 3, a + 1)
```

In [14]:
```
s.plot()+traj.plot()
```

Out[14]:



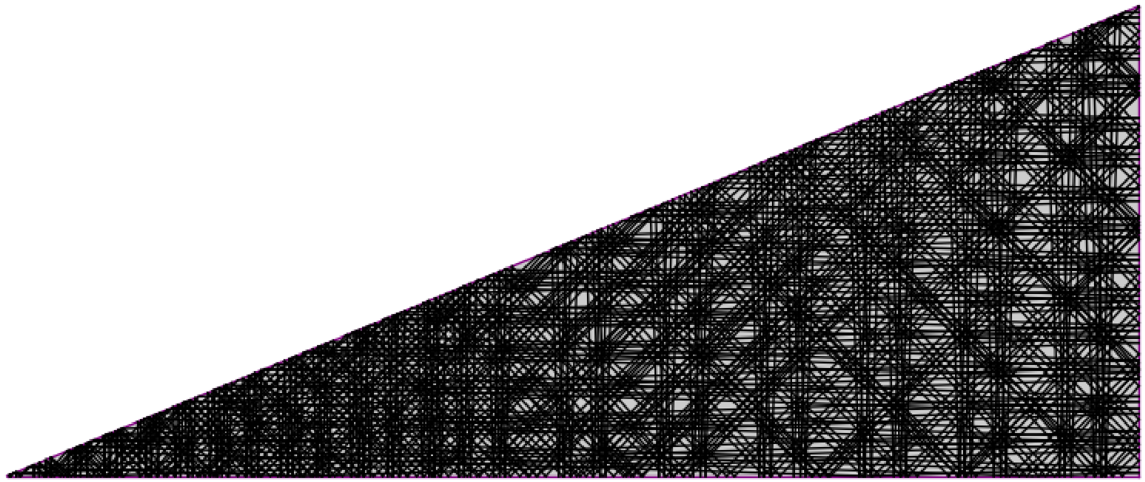
In [15]:
```
traj.is_closed()
```

Out[15]: True

In [16]:
```
v = s.tangent_vector(0,(3/4,0),(12451,13255))
traj = v.straight_line_trajectory()
traj.flow(600)
```

In [17]:
```
s.plot()+traj.plot()
```

Out[17]:



In [18]:
```
traj.is_closed()
```

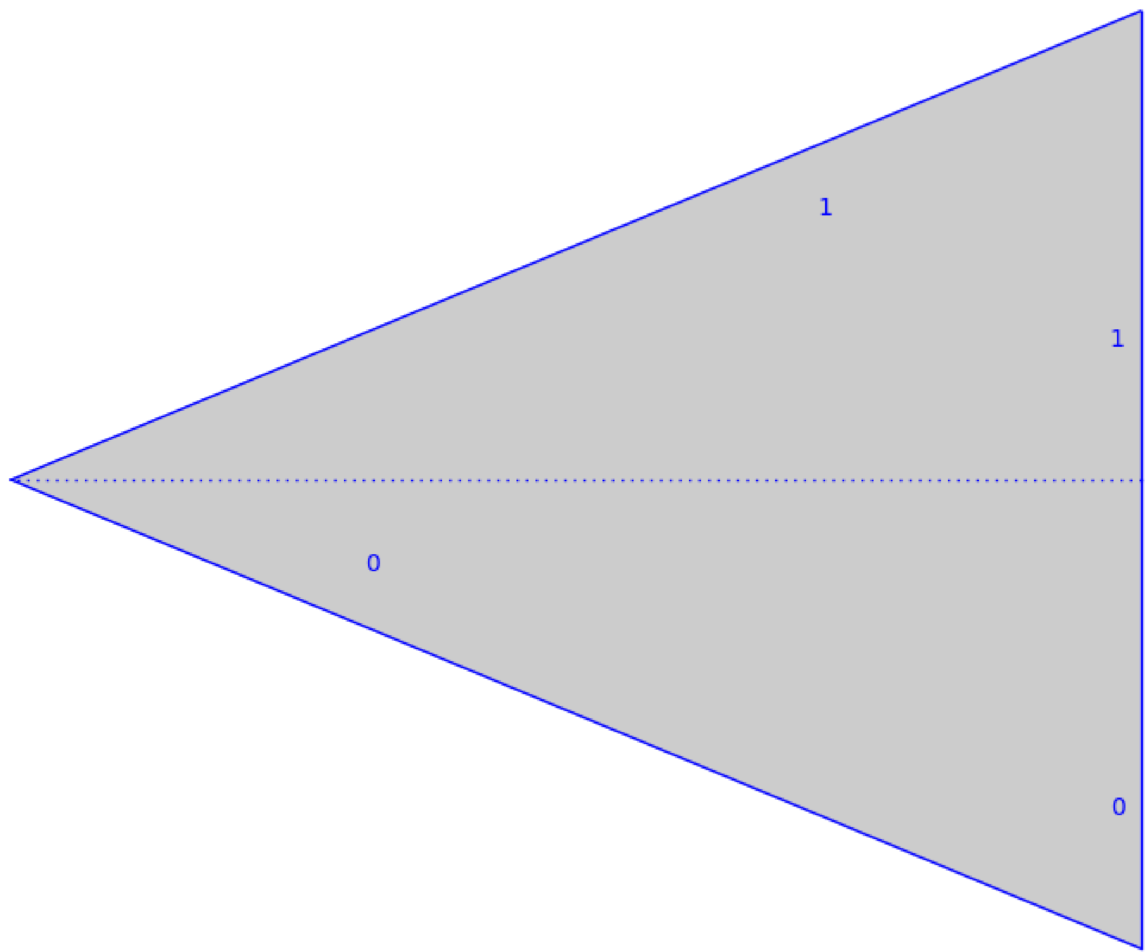
Out[18]:
```
False
```

Cone Surfaces

Secretly, the billiard table is actually a cone surface; the double of a triangle across its boundary.

In [19]:
```
s.graphical_surface().make_adjacent(0,0)
s.plot()
```

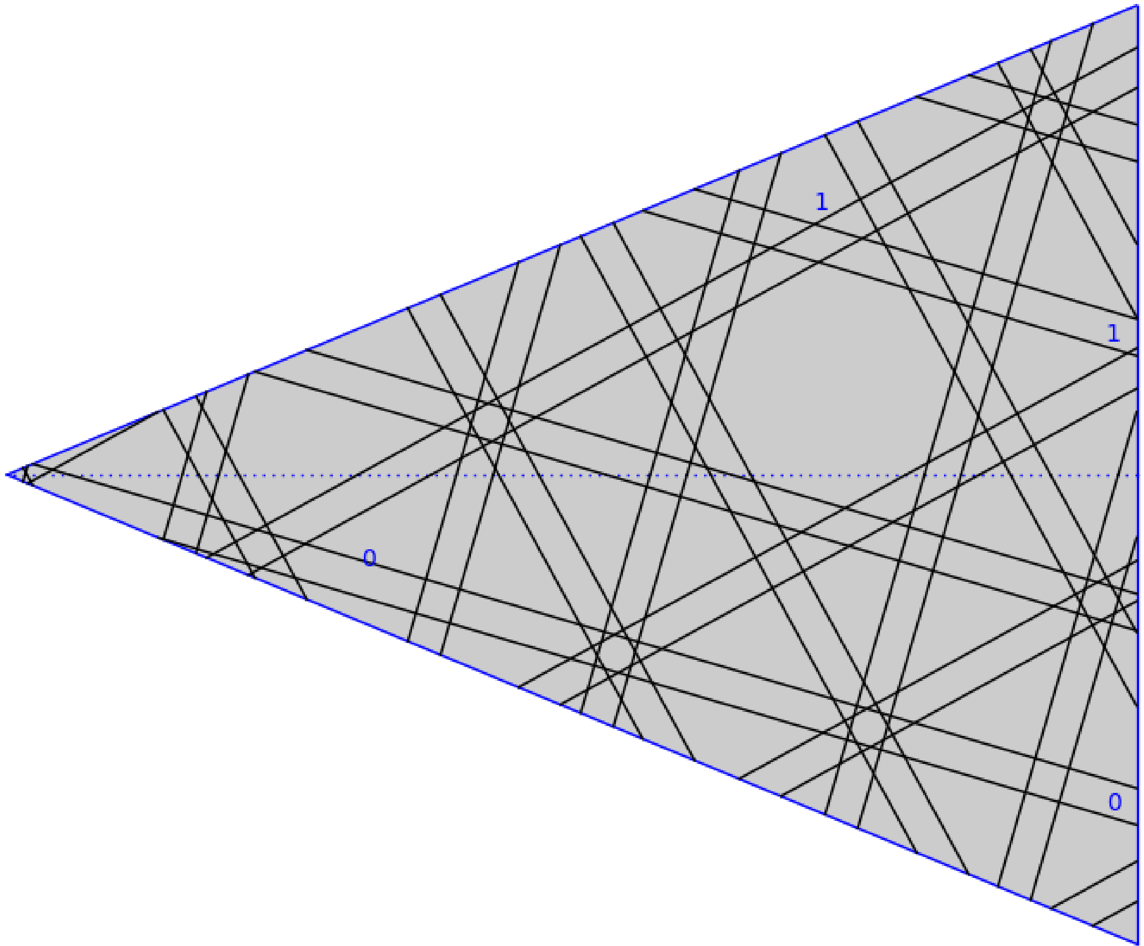
Out[19]:



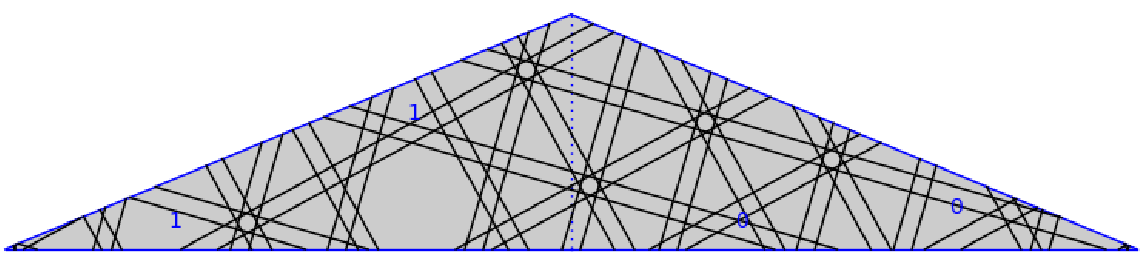
In [20]:
```
v = s.tangent_vector(0,(3/4,0),(3+a,1+a))
print(v)
traj = v.straight_line_trajectory()
traj.flow(1000)
```

```
SimilaritySurfaceTangentVector in polygon 0 based at (3/4, 0) with vector
 (a + 3, a + 1)
```

In [21]:
```
s.plot()+traj.plot()
```
Out[21]:

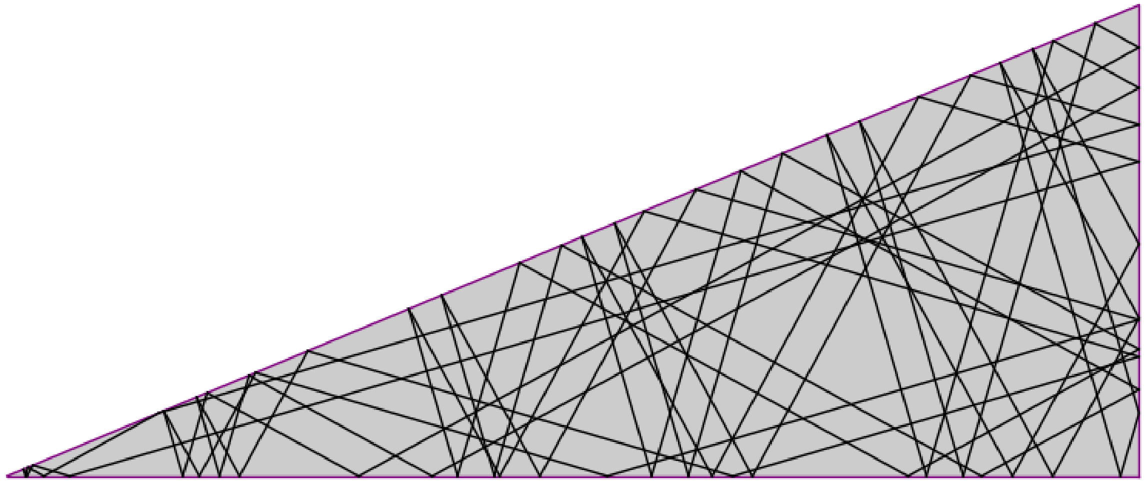


In [22]:
```
s.graphical_surface().make_adjacent(0,1)
s.plot()+traj.plot()
```
Out[22]:



In [23]:
```
s.graphical_surface().make_adjacent(0,1,reverse=True)
s.plot()+traj.plot()
```

Out[23]:



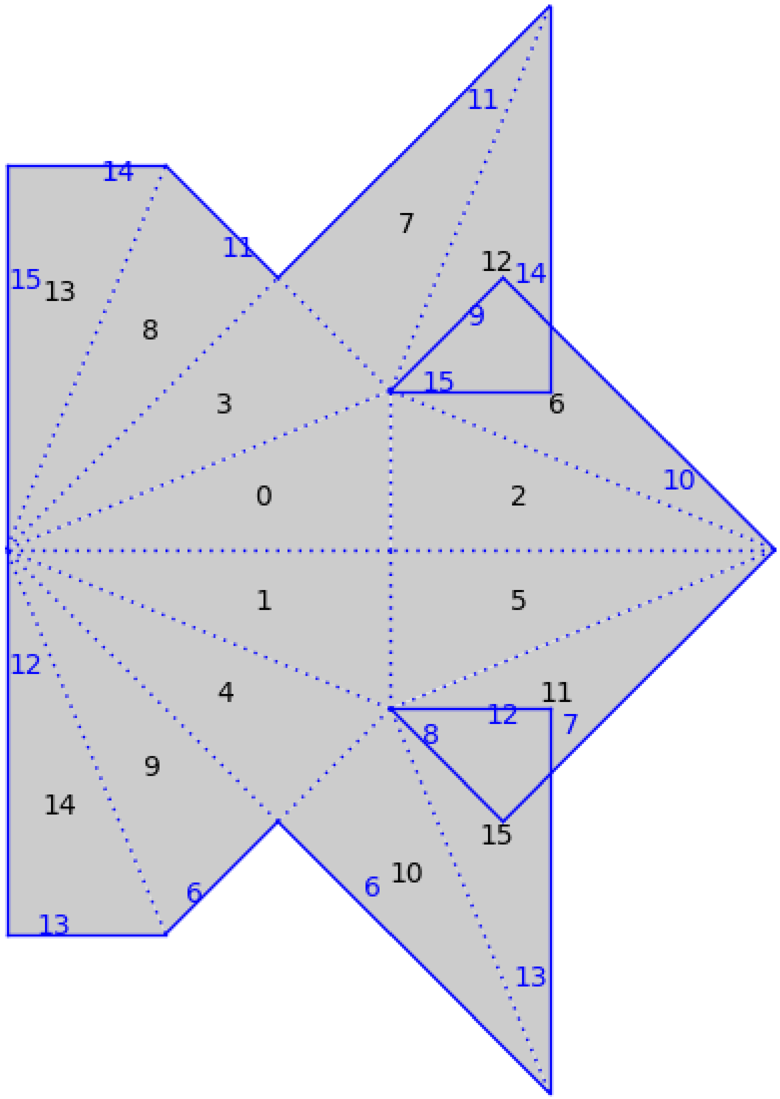
Translation Surfaces:

In [24]:
```
ts=s.minimal_translation_cover()
ts=ts.copy(relabel=True)
print(ts)
```

```
TranslationSurface built from 16 polygons
```

In [25]:
```
ts.plot()
```

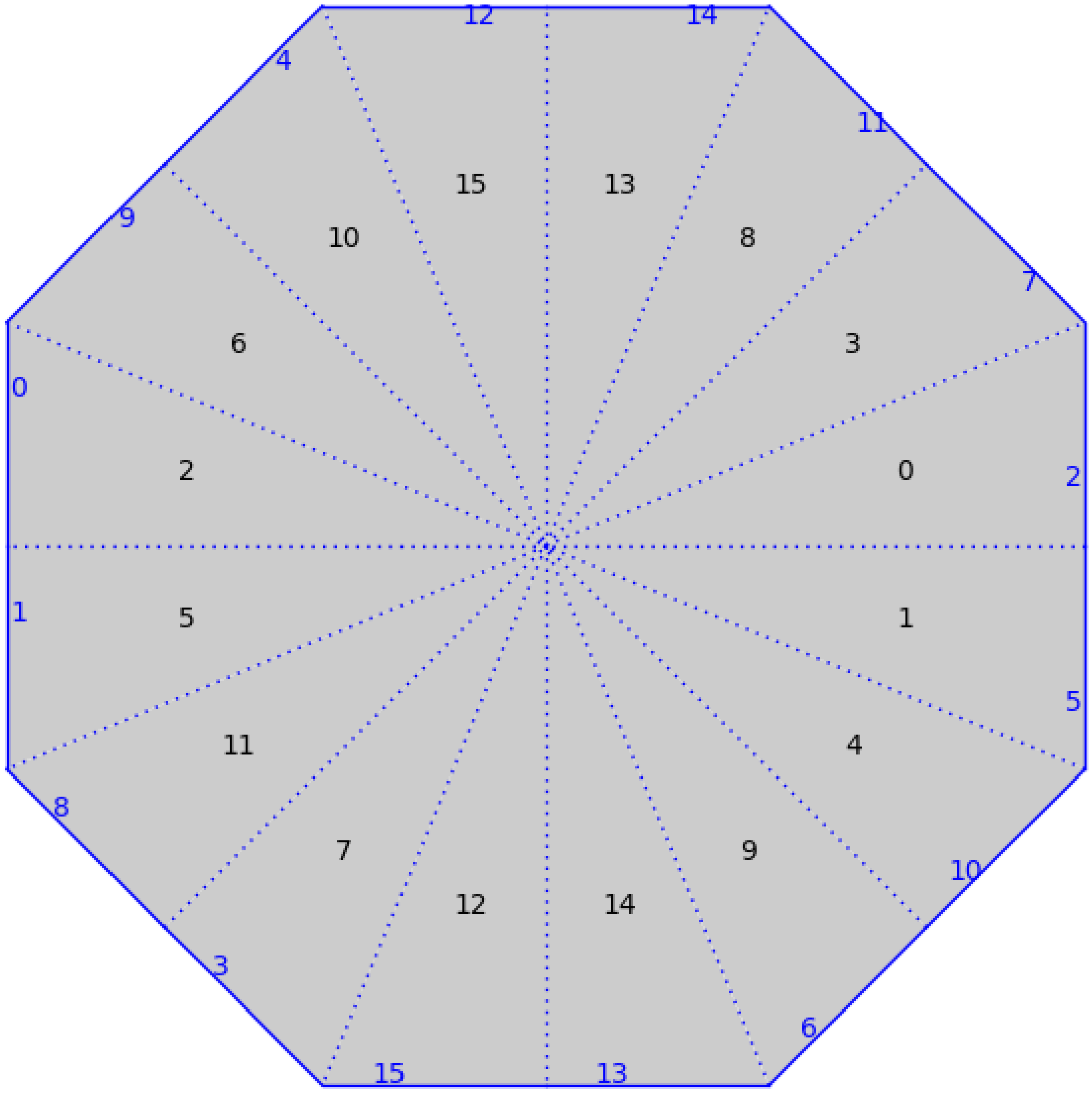
Out[25]:



In [26]:
```
adjacencies=[(13,2),(15,2),(10,2),(6,2),(2,2),(5,2),(11,2),(7,2)]
for adjacency in adjacencies:
    ts.graphical_surface().make_adjacent(*adjacency)
```

In [27]:
```
ts.plot()
```

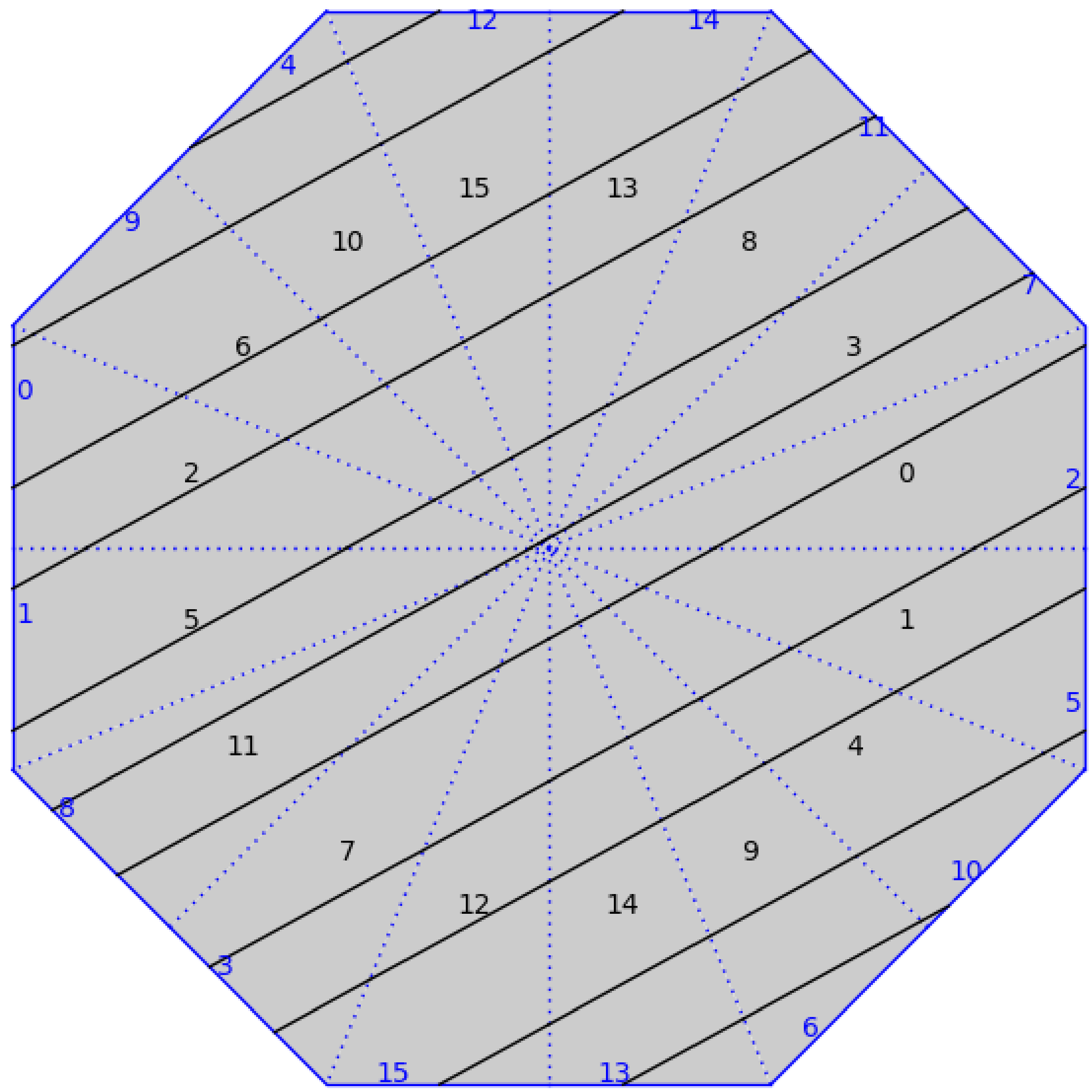
Out[27]:



In [28]:
```
v = ts.tangent_vector(0,(3/4,0),(3+a,1+a))
traj = v.straight_line_trajectory()
traj.flow(1000)
print(traj.is_closed())
```

```
True
```

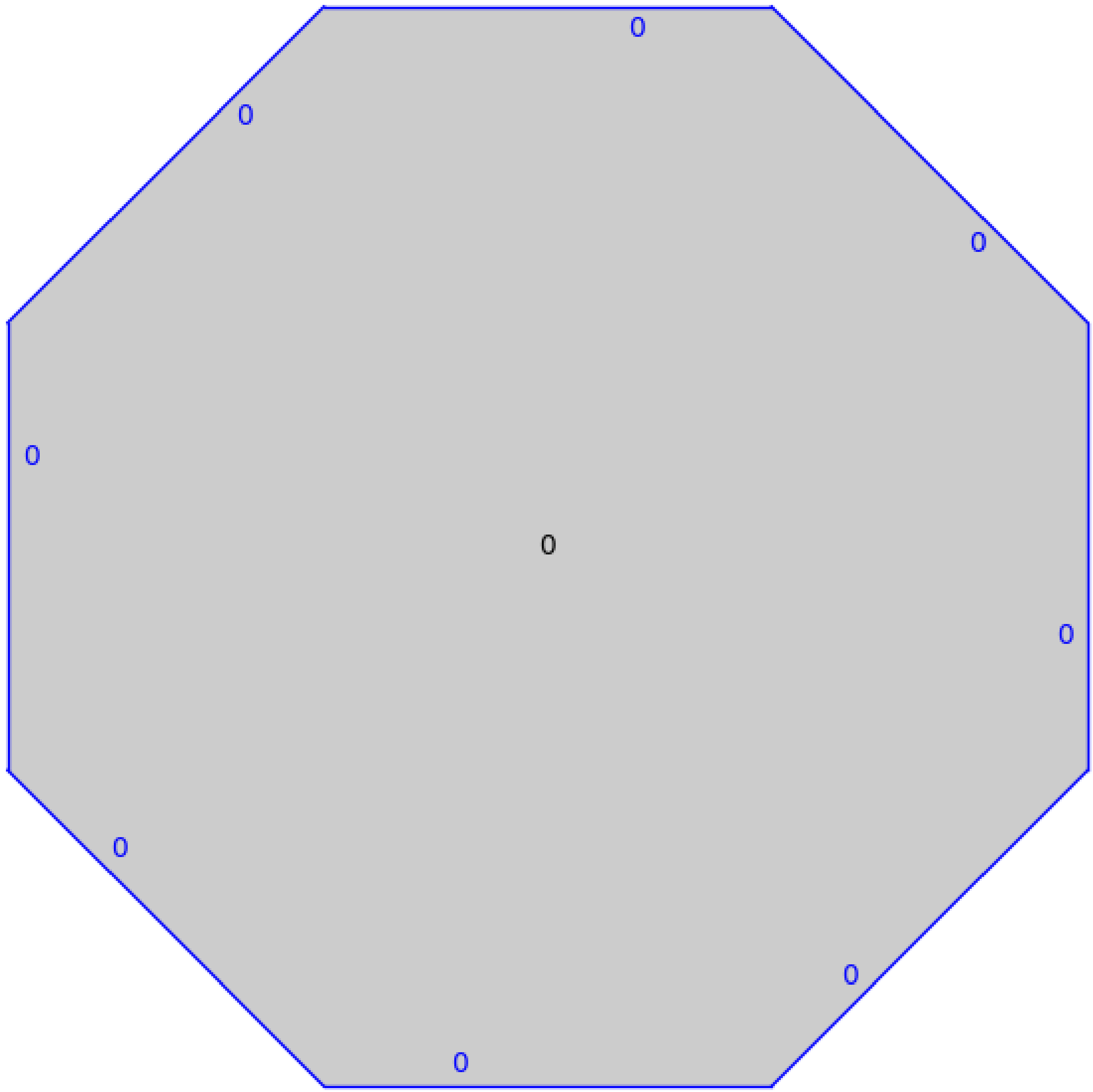
In [29]:
```
ts.plot()+traj.plot()
```
Out[29]:



In [30]:
```
ts = translation_surfaces.regular_octagon()
a=ts.base_ring().gens()[0]
print("a="+str(a)+"="+str(a.n()))
ts.plot()
```

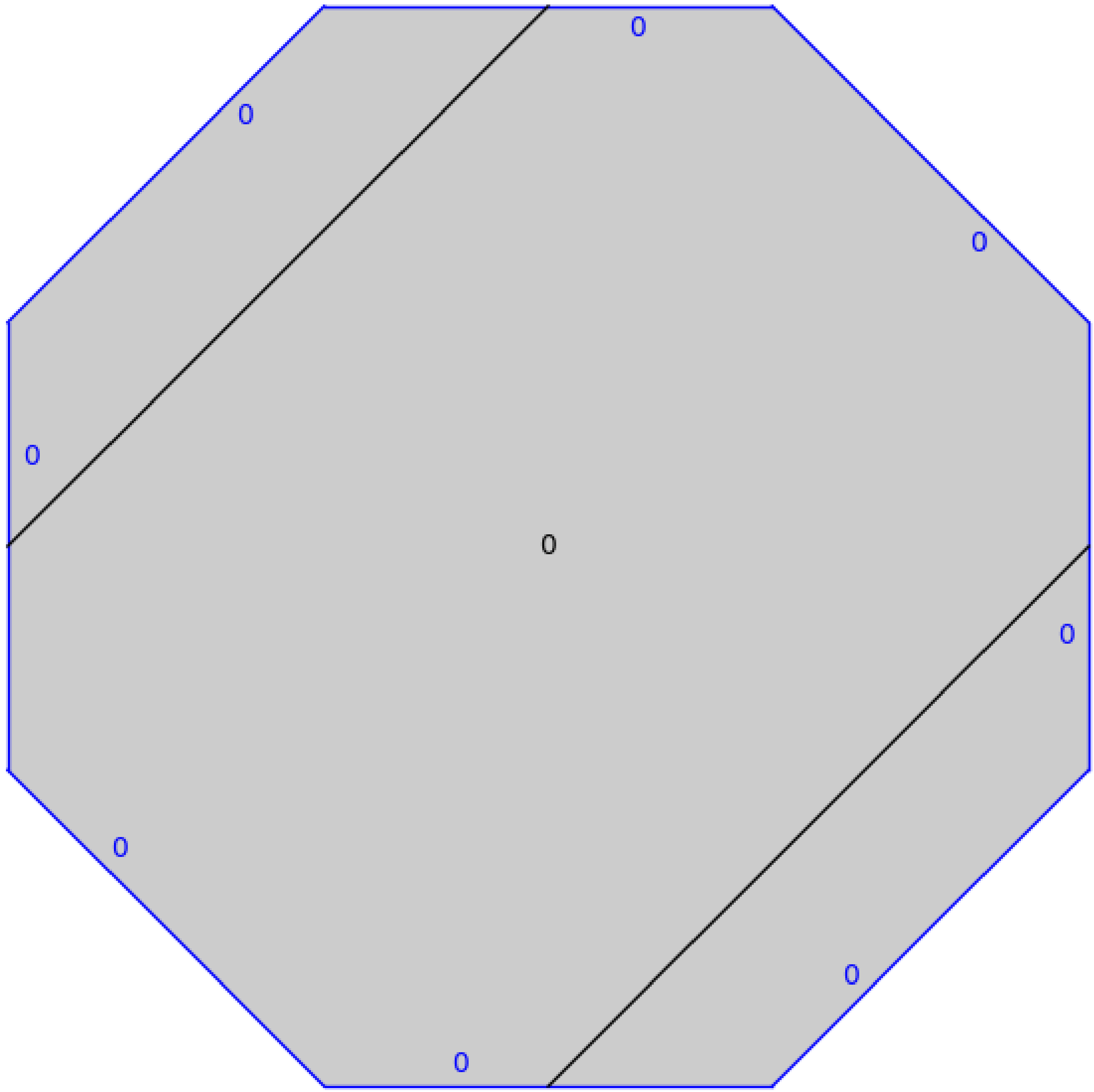
```
a=a=1.41421356237309
```

Out[30]:



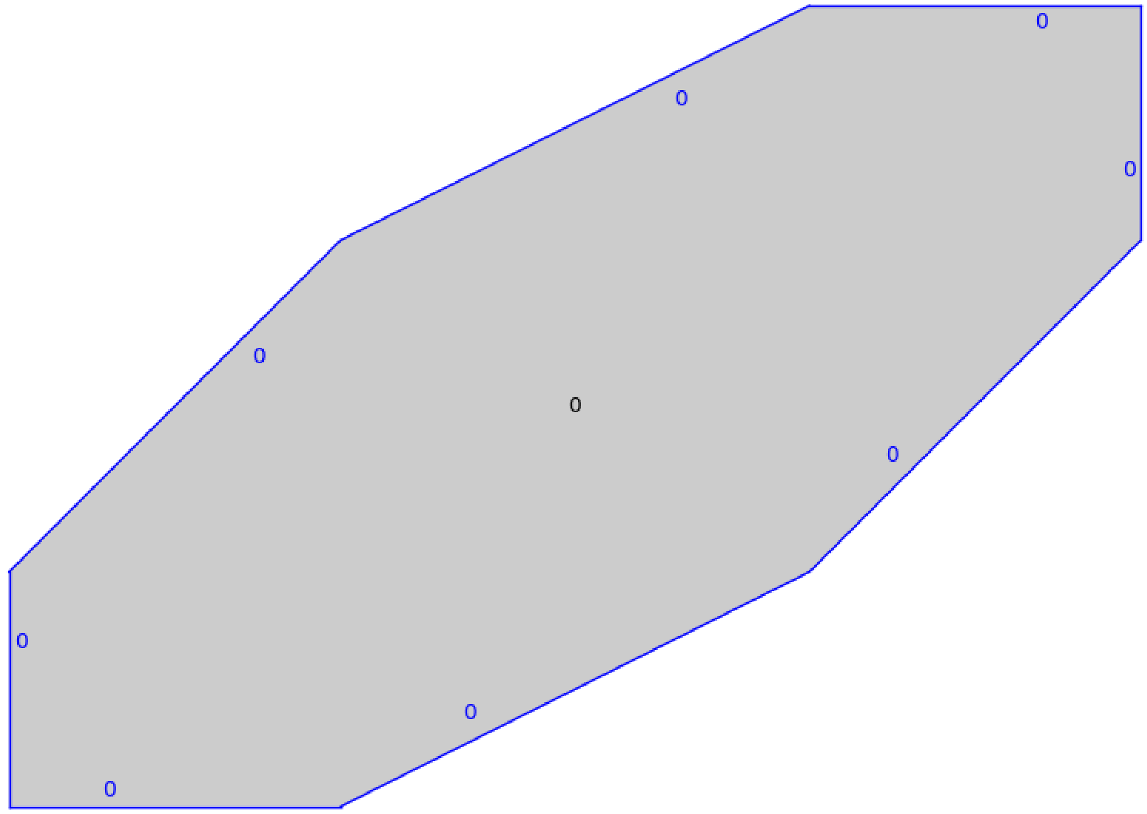
In [31]:
```
v = ts.tangent_vector(0,(1/2,0),(1,1))
traj = v.straight_line_trajectory()
traj.flow(1000)
ts.plot()+traj.plot()
```

Out[31]:

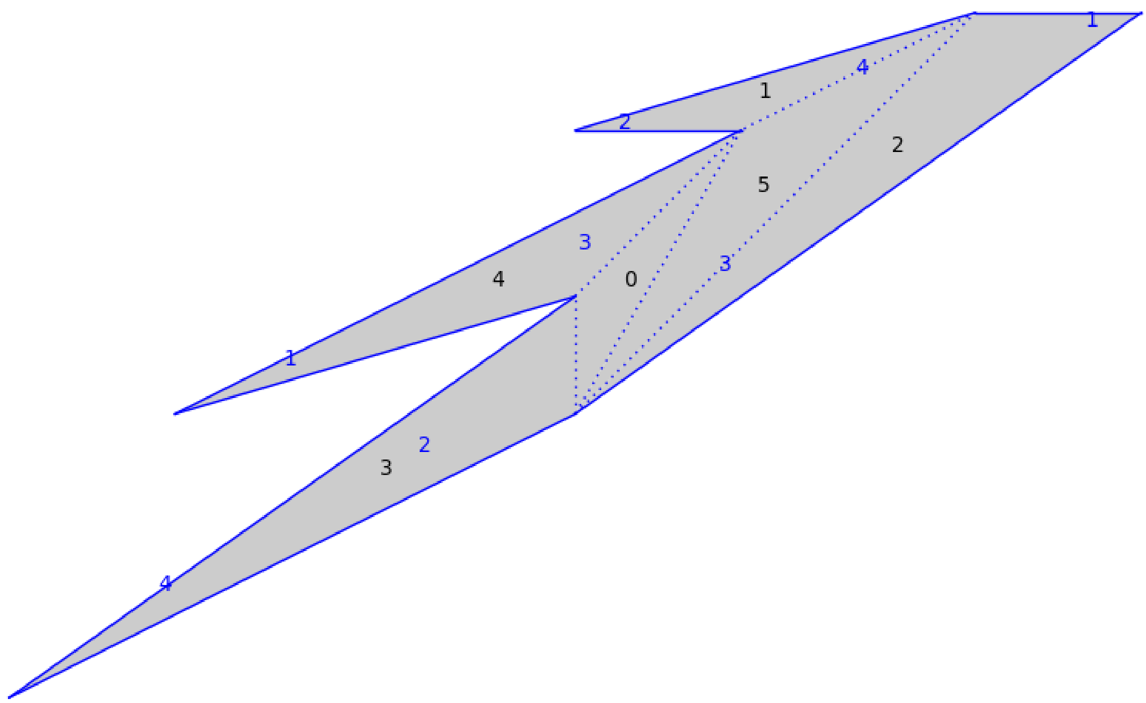


In [32]:
```
m = matrix([[1,1],[0,1]])
ts2 = m*ts
```

In [33]:
```
ts2.plot()
```
Out[33]:

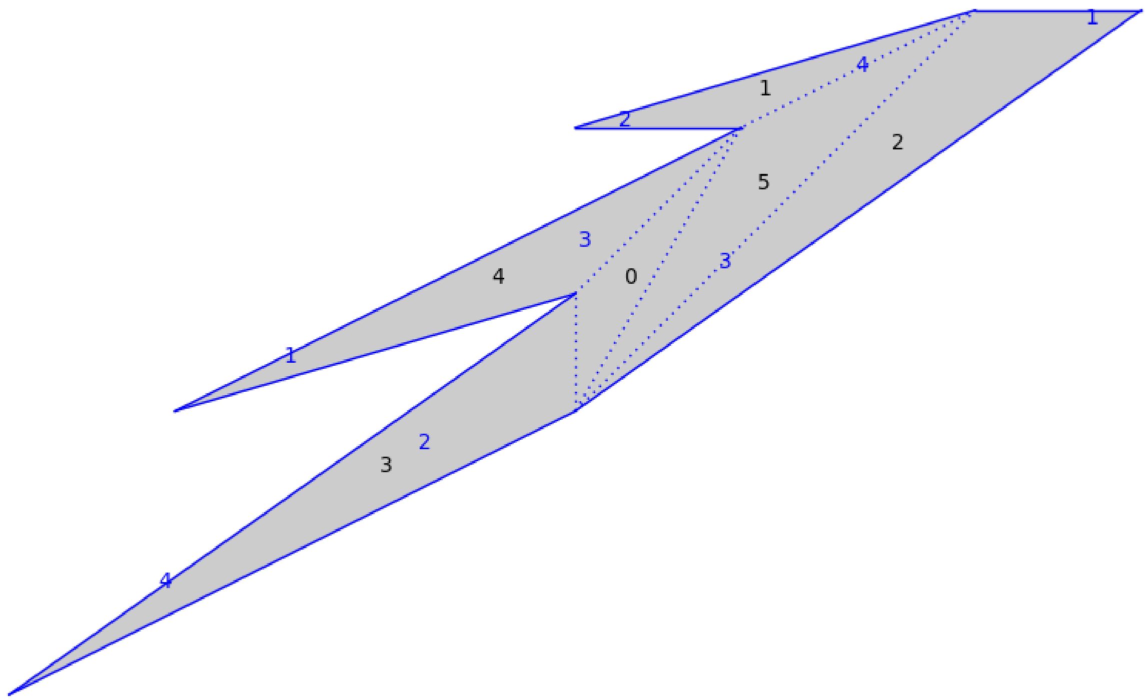


In [34]:
```
ts2=ts2.triangulate().copy(relabel=True)
ts2.plot()
```
Out[34]:



In [35]:
```
ts2.polygon(5)
```
Out[35]: Polygon: (0, 0), (-1, -1/2*a - 1), (a, 1/2*a)

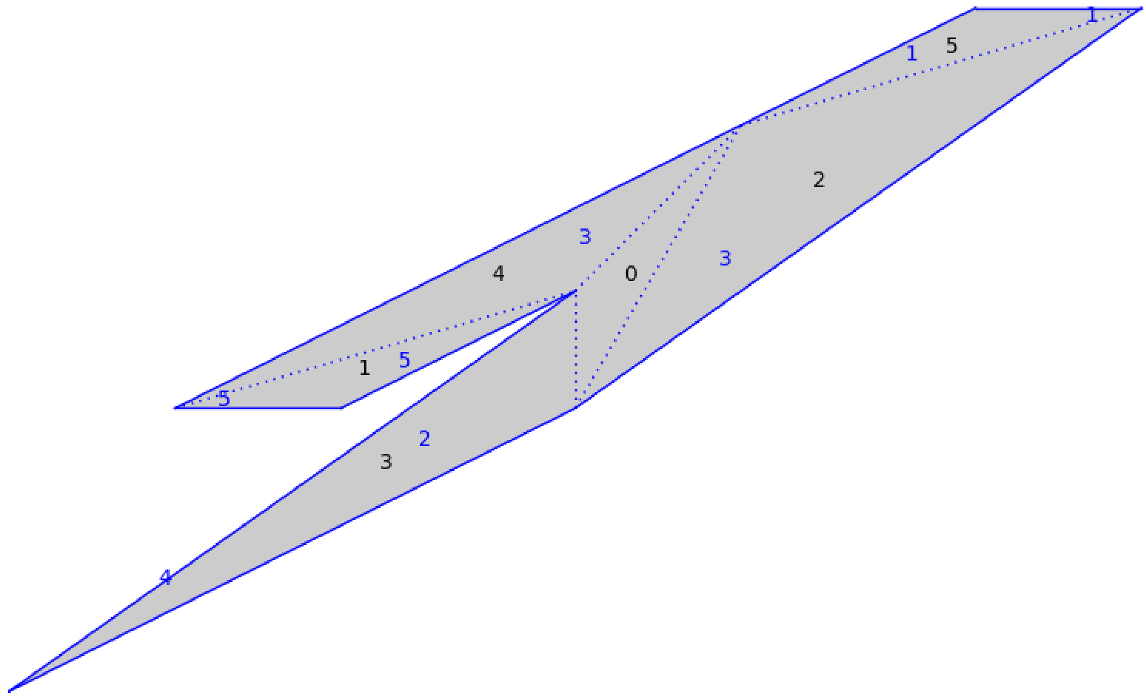
In [36]:
```
ts2.graphical_surface().make_adjacent(5,1)
ts2.plot()
```
Out[36]:



In [37]:
```
ts2=ts2.triangle_flip(5,1)
ts2.graphical_surface().make_adjacent(1,0)
```

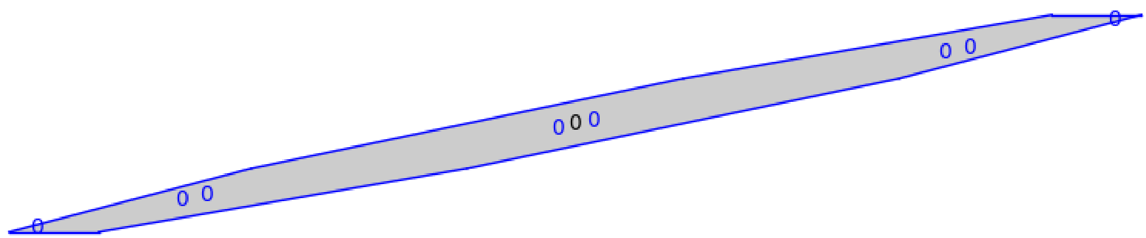
In [38]:
```
ts2.plot()
```

Out[38]:



In [39]:
```
m = matrix([[1,2+2*a],[0,1]])
ts2 = m*ts
ts2.plot()
```

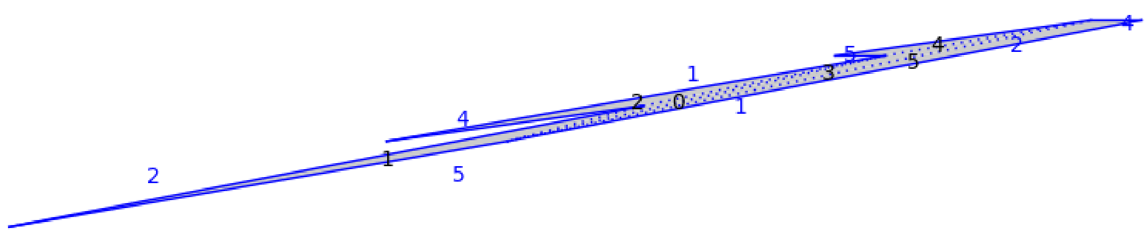
Out[39]:



In [40]:
```
ts2=ts2.triangulate().copy(relabel=True, mutable=True)
```

In [41]:
```
ts2.plot()
```

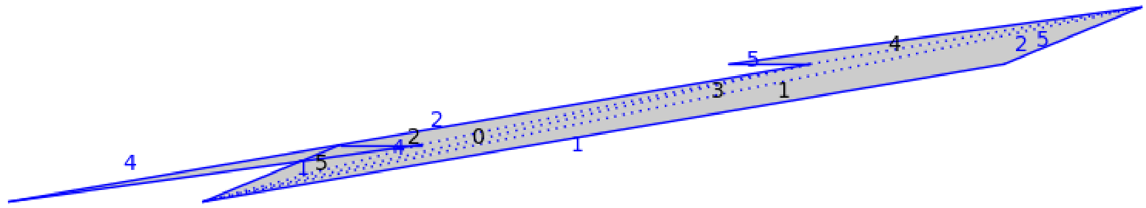
Out[41]:



In [42]:
```
print(ts2.delaunay_single_flip())
ts2.plot(cached=False)
```

True

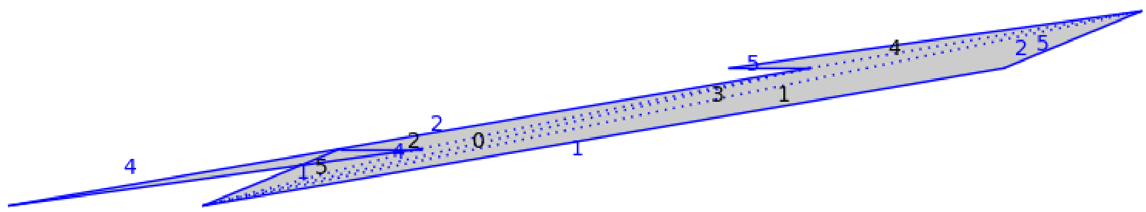
Out[42]:



In [43]:
```
print(ts2.delaunay_single_join())
ts2.plot(cached=False)
```

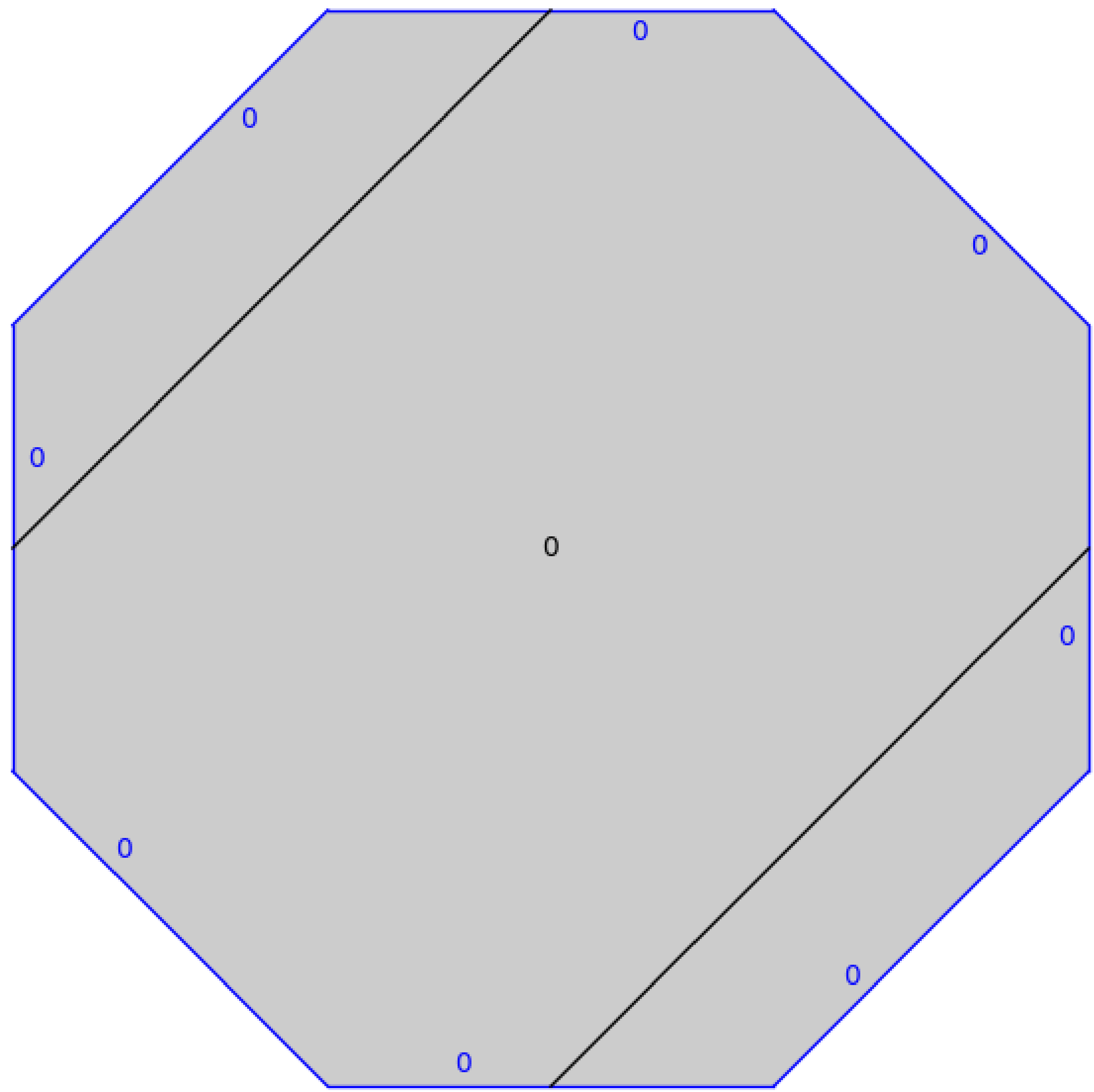
False

Out[43]:



In [44]:
```
v=ts.tangent_vector(0,(1/2,0),(1,1))
traj=v.straight_line_trajectory()
traj.flow(10)
ts.plot()+traj.plot()
```

Out[44]:



In [45]:
```
from flatsurf.geometry.half_dilation_surface import GL2RMapping
```

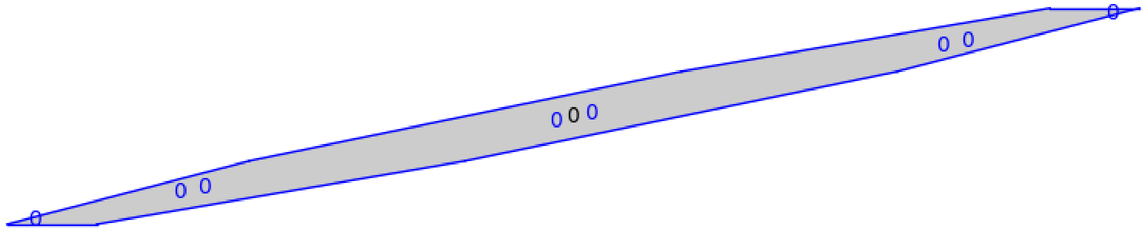
In [46]:
```
m
```

Out[46]:
```
[      1 2*a + 2]
[      0       1]
```

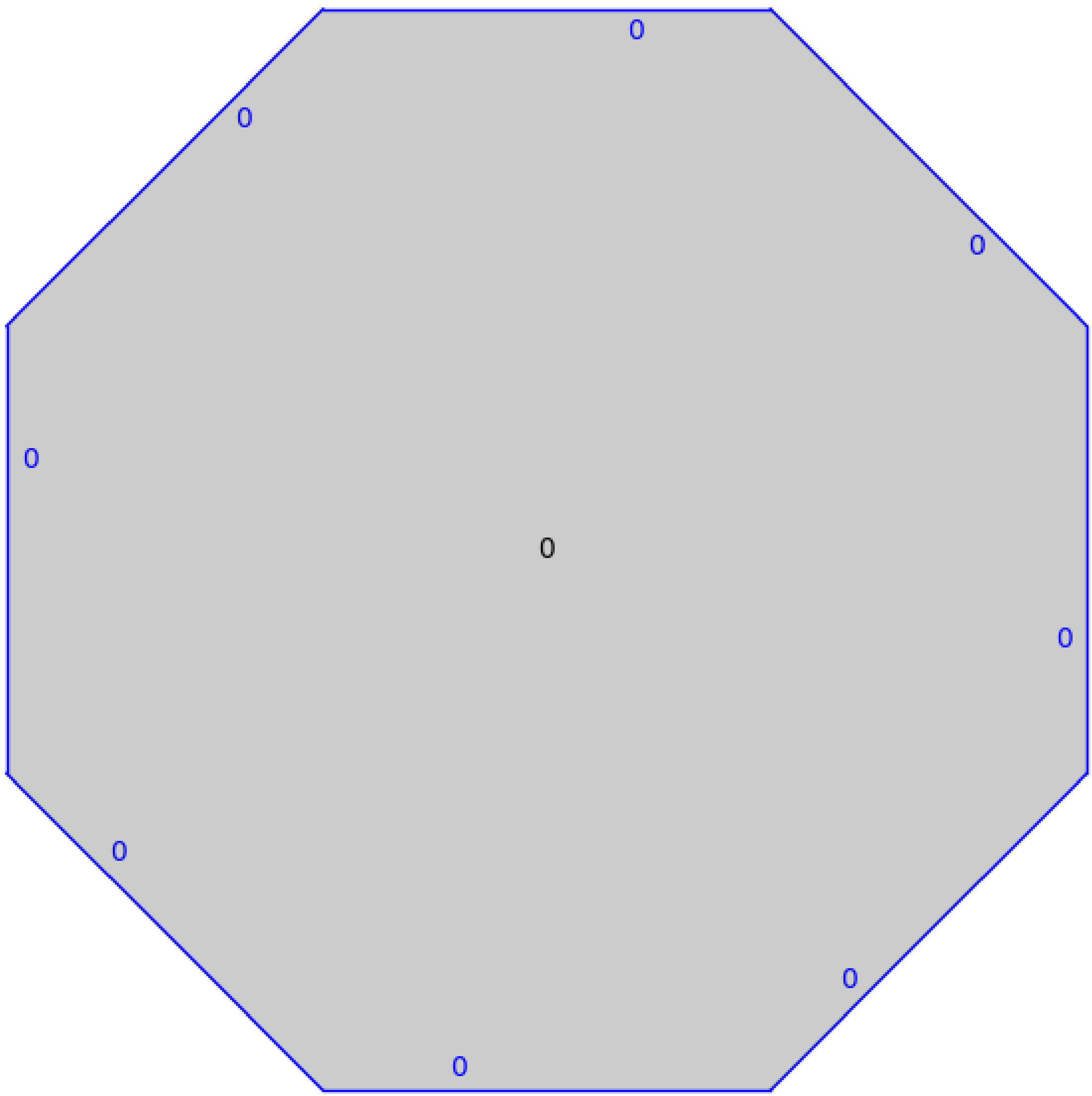
In [47]:
```
map1=GL2RMapping(ts,m)
```

In [48]:
```
ts2=map1.codomain()
```

In [49]:
```
ts2.plot()
```
Out[49]:



In [50]:
```
map2=ts2.canonicalize_mapping()
map2.codomain().plot()
```
Out[50]:

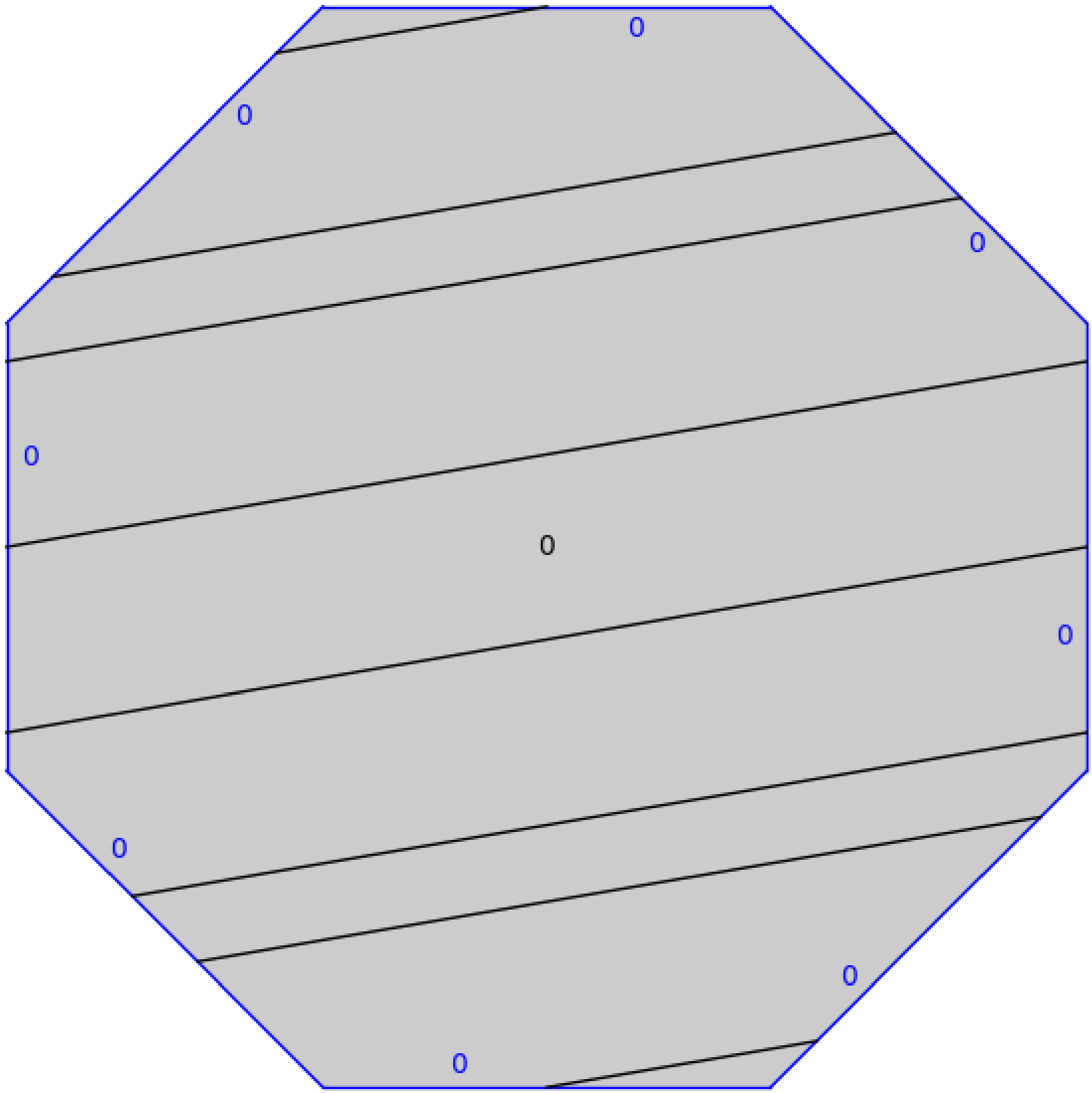


In [51]:
```
Map = map2*map1
```

In [52]:
```
w=Map.push_vector_forward(v)
```

In [53]:
```
traj2=w.straight_line_trajectory()
traj2.flow(100)
Map.codomain().plot()+traj2.plot()
```

Out[53]:



In [54]:
```
traj2.segments()[0]
```

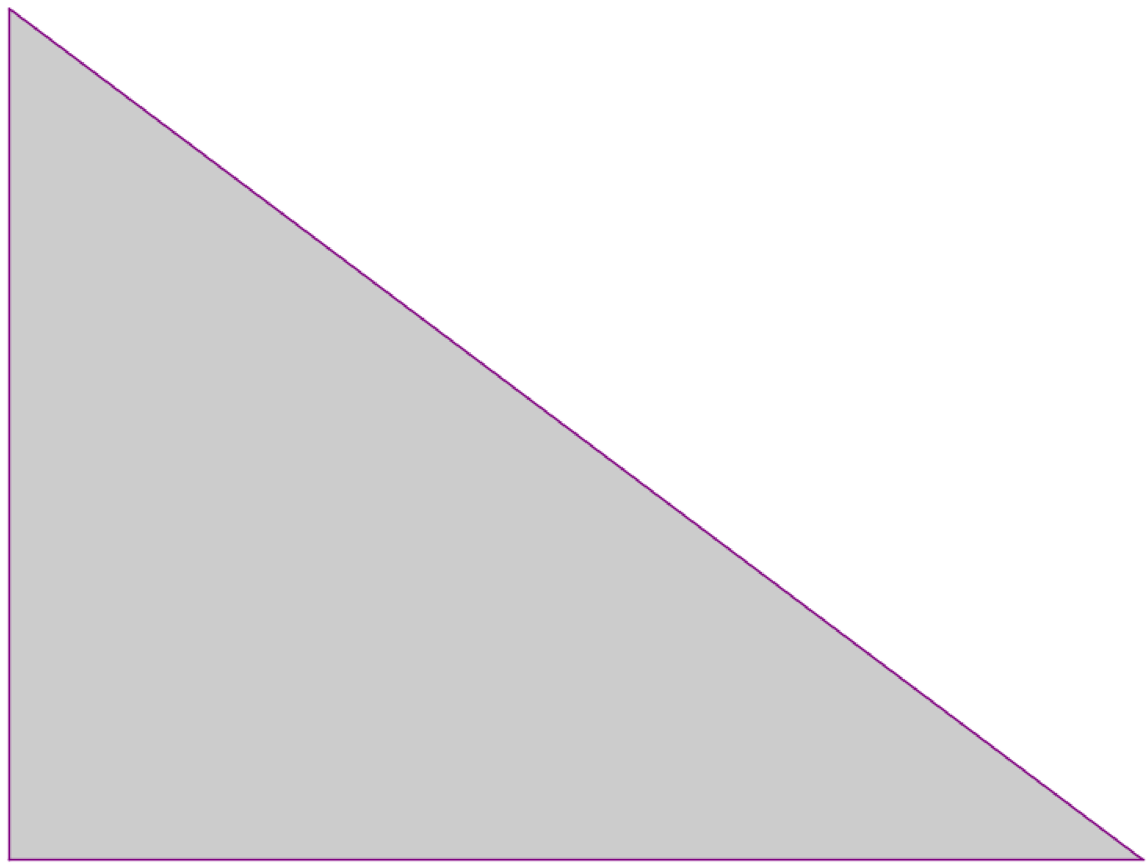
Out[54]:
```
Segment in polygon 0 starting at (1/2, 0) and ending at (1/4*a + 3/4, 1/4*a
 - 1/4)
```

Infinite surfaces

We build the billiard table for the 3-4-5 right triangle.

In [55]:
```
from flatsurf import *
p = polygons(vertices=[(0,0),(4,0),(0,3)])
s = similarity_surfaces.billiard(p)
s.plot()
```

Out[55]:

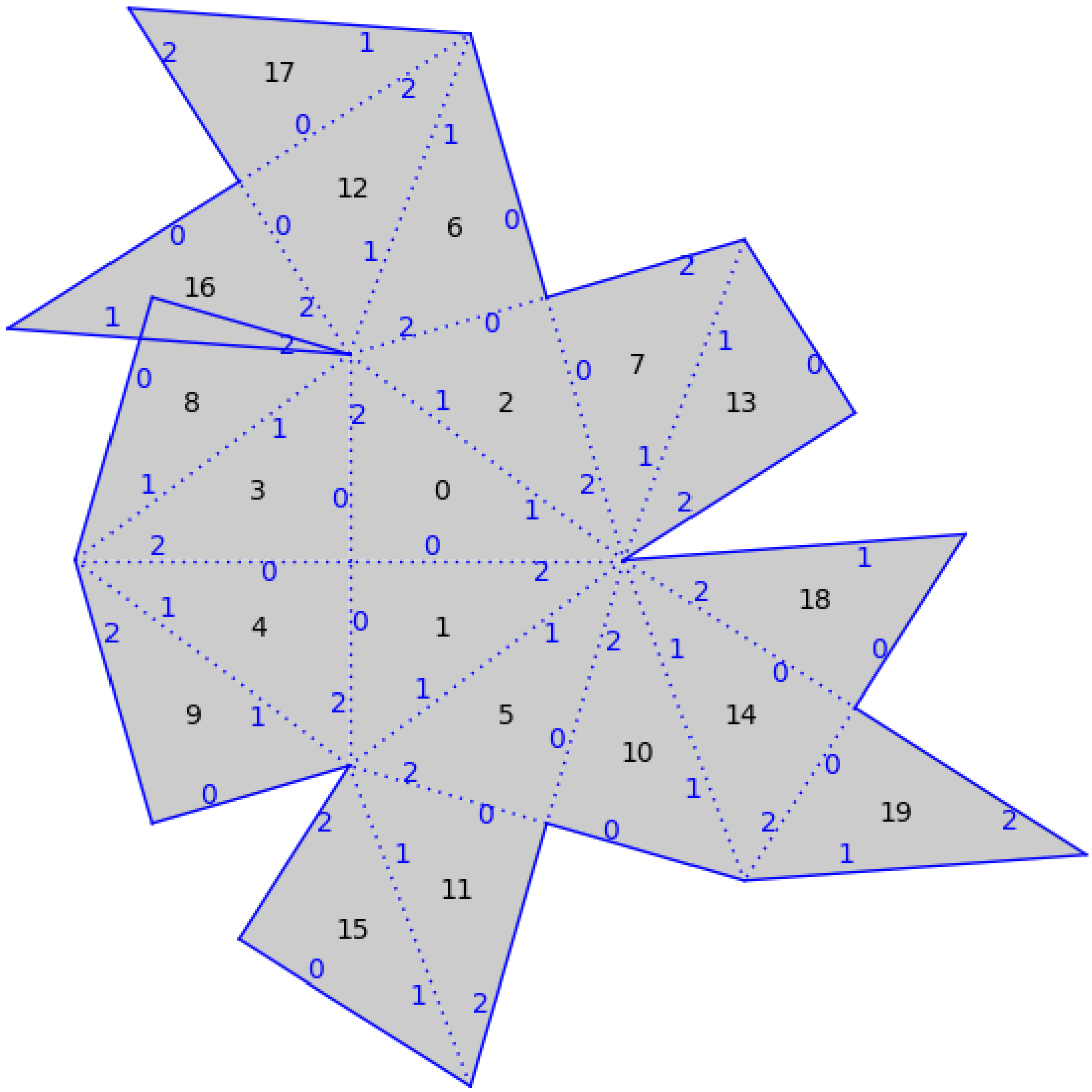


In [56]:
```
ts=s.minimal_translation_cover()
ts=ts.copy(relabel=True)
```

```
Warning: Could be indicating infinite surface falsely.
```

In [57]:
```
gs = ts.graphical_surface(polygon_labels=False)
gs.make_all_visible(limit=19)
gs.process_options(polygon_labels=True, edge_labels="number")
gs.plot()
```

Out[57]:

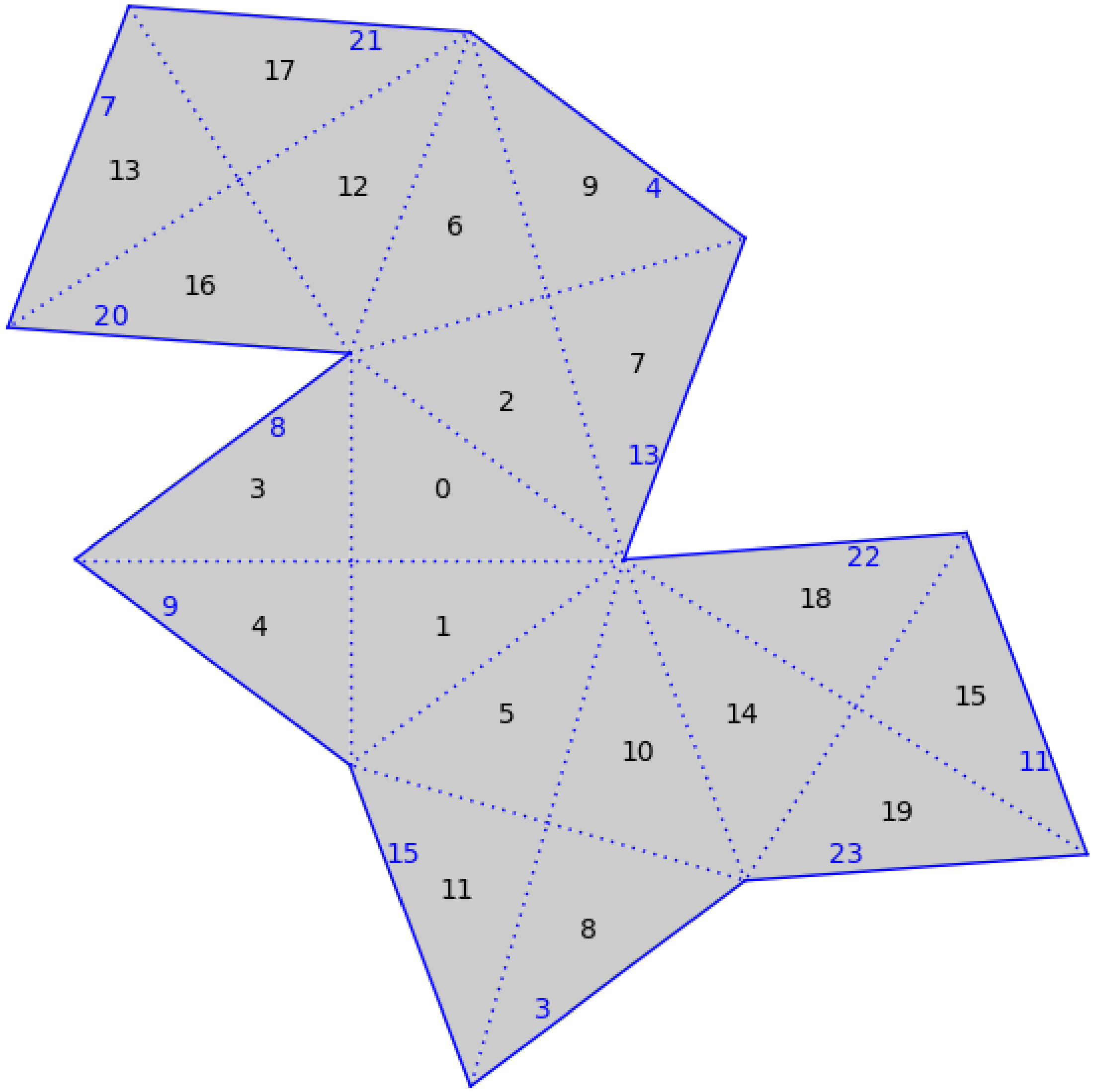


In [58]:
```
gs=ts.graphical_surface()
```

In [59]:
```
gs.make_adjacent_and_visible(18,0)
gs.make_adjacent_and_visible(10,0)
gs.make_adjacent_and_visible(6,0)
gs.make_adjacent_and_visible(16,0)
gs.process_options(polygon_labels=True, edge_labels="gluings")
```

In [60]:
```
ts.plot()
```

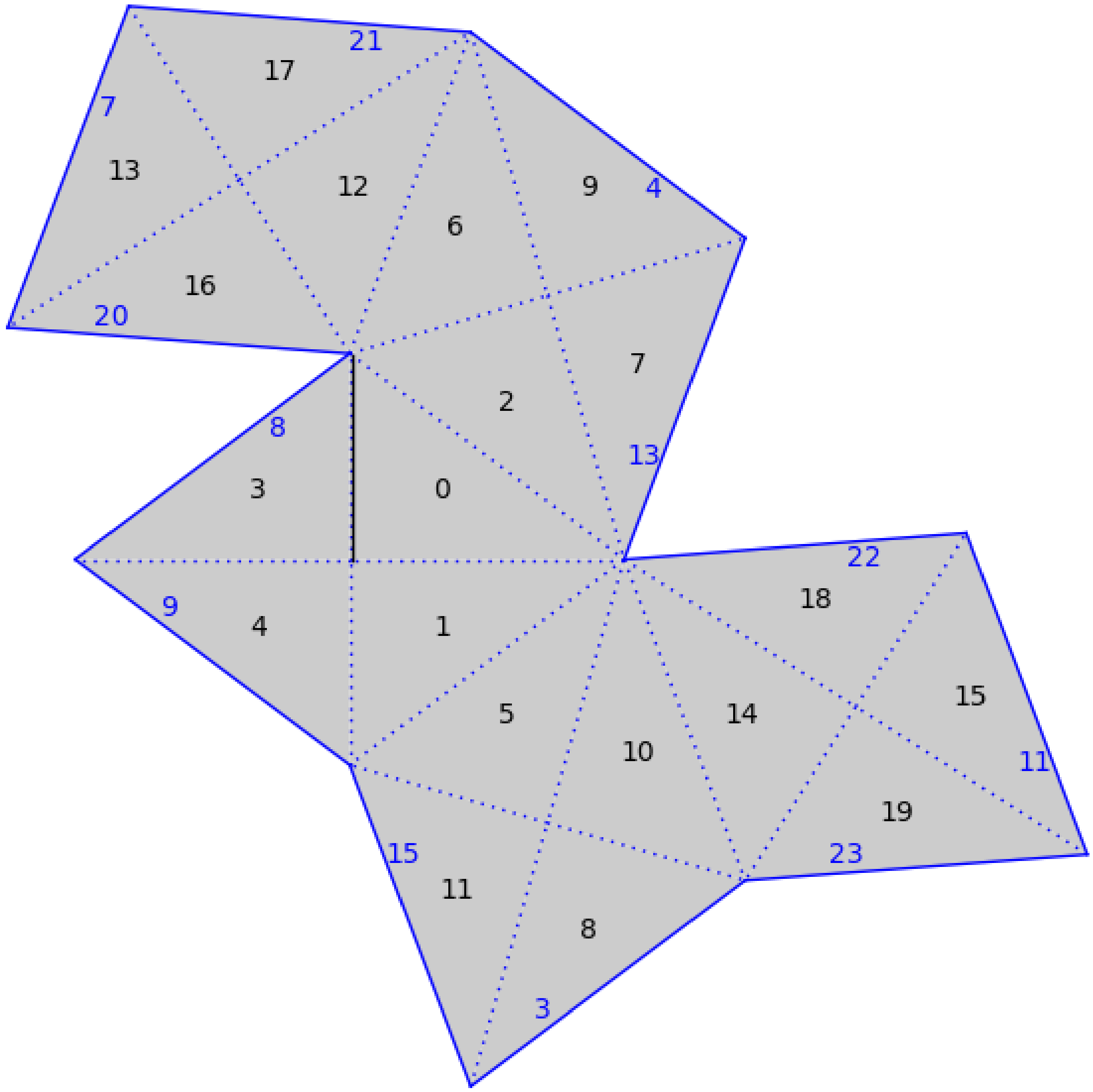
Out[60]:



In [61]:
```
v=ts.tangent_vector(0,(1/25,0),(0,1))
traj=v.straight_line_trajectory()
```

In [62]:
```
gs.plot()+traj.plot()
```

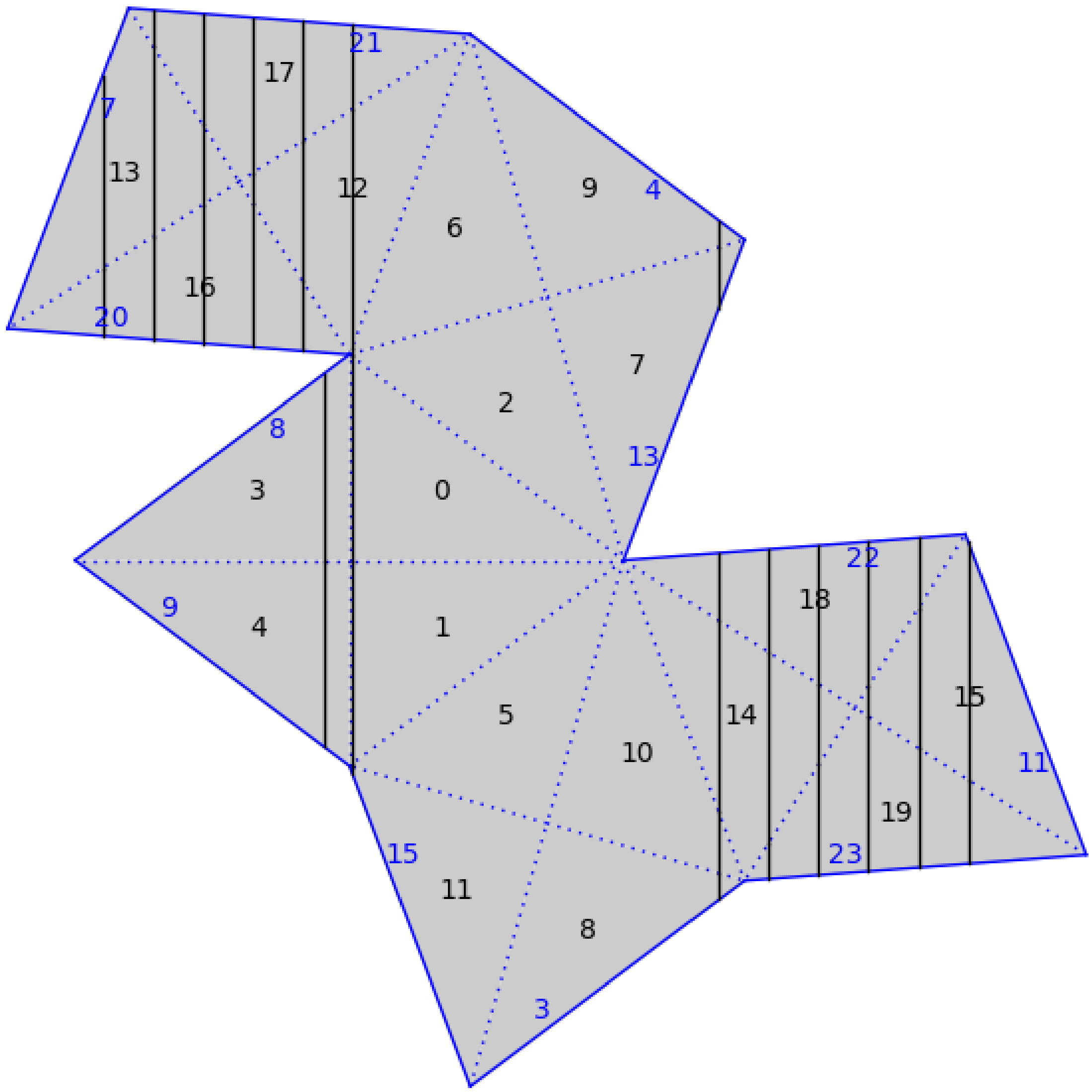
Out[62]:



In [63]:
```
traj.flow(1000)
print(traj.is_closed())
gs.plot()+traj.plot()
```

```
True
```

Out[63]:



In [64]:
```
gs.process_options(polygon_labels=True, edge_labels="number")
```

In [65]:
```
gs.plot()
```

Out[65]: