

# Posets and words in Sage-Combinat

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Thus, words arise in several areas of mathematics and the sciences:

- *word problem* in semigroup and group theory;
- *permutations as words* in combinatorics;
- *automatic sequences* in number theory;
- *DNA* in biology;
- *words* in linguistics;
- *etc.*

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Examples: want efficient algorithms and data structures for

- searching text;
- pattern recognition;
- inferring combinatorial, probabilistic and statistical properties;
- counting distinct factors;
- storing and retrieving factors;
- factorizations (Lyndon, Crochemore, ...);
- .....

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- I give a very enthusiastic talk about it when I get back.
- Others get excited too!

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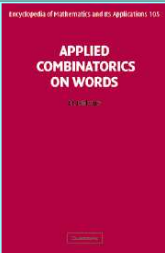
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- Sept. 2008: People at MLV also want to get in on the action; we'll be discussing more about what should be included soon.



# Pre-existing software?

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# Pre-existing software?

## Applied Combinatorics on Words: Implementation of algorithms



### Algorithms on words

A set of [computer programs in Java](#) for the algorithms of Chapter 1 is available in a preliminary form. They can be freely copied and used with the mention of their origin. The idea is to present an illustration of a possible effective implementation rather than fine tuned optimal software. No guarantee at all is given for correctness. A [documentation](#) is in progress.

### Structures for indexes

[Computer programs in Java and C](#) for the algorithms of Chapter 2 and for other text processing algorithms are available.

### Statistical natural language processing

Programs for the algorithms of this chapter are available at:

<http://www.research.att.com/sw/tools/fsm>

<http://www.research.att.com/sw/tools/grm>

<http://www.research.att.com/sw/tools/dcd>

### Statistics on words with applications to biological sequences

Computations of words with exceptional frequency in DNA were performed with programs available at: <http://www-mig.jouy.inra.fr/ssb/rmes/>

### Periodic structures in words

Concerning this chapter, principal algorithms have been implemented in the mreps software <http://www.loria.fr/mreps/>.

What does sage-words do?

Demo

What's next for sage-words?

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- Cythonize!

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- Very important: Need to be able to take objects (say, permutations) and turn them into posets *easily*.