Introduction to Sage

Sébastien Labbé

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Groupe d'utilisateurs de Sage de Paris Observatoire de Paris November 6th 2013 "You can read Sylow's Theorem and its proof in Huppert's book in the library ...then you can use Sylow's Theorem for the rest of your life free of charge, but for many computer algebra systems license fees have to be paid regularly

With this situation two of the most basic rules of conduct in mathematics are violated: In mathematics information is passed on free of charge and everything is laid open for checking."

—J. Neubüser (1993) (il a fondé GAP en 1986)

Why You Do Not Usually Need to Know about Internals

« You should realize at the outset that while knowing about the internals of Mathematica may be of intellectual interest, it is usually much less important in practice than you might at first suppose. [...] For the internals of Mathematica are quite complicated, [...], it is usually extremely difficult to reach a reliable conclusion about how the detailed implementation of this algorithm will actually behave in particular circumstances. »

-Mathematica Tutorial

 $http://reference.\ wolfram.\ com/mathematica/tutorial/$ $\ Why You Do Not Usually Need To Know About Internals.\ html$

"I think, fundamentally, open source does tend to be more stable software. It's the right way to do things. I compare it to science versus witchcraft.

In science, the whole system builds on people looking at other people's results and building on top of them.

In witchcraft, somebody had a small secret and guarded it—but never allowed others to really understand it and build on it."

—Linus Torvalds

Outline

- 1 History of Sage
- 2 Sage
- 3 Community
- Some functionalities

Plan

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Before Sage

Existing commercial softwares (general purpose):

- Maple, Waterloo Maple Inc., Maplesoft, since 1985.
- Matematica, Wolfram Research, since 1988.
- Magma, University of Sydney, since 1990
- Matlab, MathWorks, since 1989

Free existing softwares (specific purpose):

- Maxima, W. Schelter et coll., since 1967: symbolic operations
- GAP, GAP Group, since 1986 : computational group theory
- Singular, U. of Kaiserslautern, since 1984 : polynomial comput.
- PARI/GP, since 1985 : number theory computations

- 1999-2005. William Stein write over 25000 lines of Magma code for its own research, but had some serious concerns :
 - Magma do not allow users to define their own types (or classes)
 - No exception handling
 - No "eval statements"
 - No way for users to write compiled code
 - Magma is closed source
 - Magma is not free

« I sat down on that park bench, and realized what a dangerous path I was taking in giving up so much freedom so early in my career. I resolved at that moment not to do it. At that moment I started designing what would eventually become Sage. »

— William Stein

Mathematical Software and Me : A Very Personal Recollection, 2009



- Jan. 2005. William Stein creates Sage.
- Feb. 2005. Sage version 0.1 : gathers 5 software : PARI, Maxima, Python, Singular et GAP.
- David Kohel in Australia, David Joyner, in Maryland, joined the project.
- Dec. 2005. William receives an email from Magma leader.

Date: Mon, 19 Dec 2005 16:54:09 -0800

From: "John Cannon" <john@maths.usyd.edu.au>

Subject: Magma calculator

William,

This is to formally advise you that your permission to run a general-purpose calculator based on Magma ends on Dec 31, 2005. This was originally set up at your request so students in your courses at Harvard could have easy access to Magma.

Please confirm receipt of this letter. Wishing you a happy Christmas, John

« This single email seriously scared me. Though I was working on Sage very hard for nearly a year at this point, I honestly didn't then expect Sage to really be able to replace Magma for me. [...] What John made crystal clear to me was that he could destroy my entire longterm plans on a whim. I looked around for other options, and there just weren't any. Sage had to succeed. »

— William Stein

Mathematical Software and Me : A Very Personal Recollection, 2009

History of Sage

- Feb. 2006. Sage version 1.0 is released; first "annual" Sage Days are organised: Sage Days 1.
- Dec. 2007. Sage gets slashdoted :



 2008 Mupad-Combinat group (lead by Nicolas Thiéry et Florent Hivert) switches to Sage: Sage-Combinat

History of Sage

Year	# Sage Days	Releases	Functions	Contributors
2005	0	120		3
2006	2	90		
2007	4	60	17947	
2008	8	23	22034	≈ 100
2009	9	14	23763	
2010	13	16	26592	219
2011	14	5	28462	235
2012	15	9	30508	249
2013	15	6	33772 (5.12)	260

Plan

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Mission

The Sage Project aims to create a viable high-quality and <u>open-source</u> alternative to Magma, Maple, Mathematica, Matlab and MuPAD, and to foster a friendly community of users and developers.

- The freedom to use the software (it is free).
- The freedom to read the source code.
- The freedom to improve the software.
- The freedom to redistribute the modified software to anyone.

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Sharing data in *astronomy* is not new

In the case of IUE (International Ultraviolet Explorer) spectra (1978-1996), there are five times more publications from data retrieved in the archive than from the selected observing teams (Wamsteker, Griffin, 1995).

— Françoise Genova, Observatoire de Strasbourg

Sage is . . .

a distribution of softwares

These softwares are included in Sage :

ATLAS Automatically Tuned Linear Algebra Software BLAS Basic Fortan 77 linear algebra routines Bzip2 High-quality data compressor Cddlib Double Description Method of Motzkin Common Lisp Multi-paradigm and general-purpose programming lang. CVXOPT Convex optimization, linear programming, least squares Cython C-Extensions for Python F2c Converts Fortran 77 to C code Flint Fast Library for Number Theory FpLLL Euclidian lattice reduction FreeType A Free, High-Quality, and Portable Font Engine

These softwares are included in Sage:

G95 Open source Fortran 95 compiler GAP Groups, Algorithms, Programming GD Dynamic graphics generation tool Genus2reduction Curve data computation Gfan Gröbner fans and tropical varieties Givaro C++ library for arithmetic and algebra GMP GNU Multiple Precision Arithmetic Library GMP-ECM Elliptic Curve Method for Integer Factorization **GNU TLS** Secure networking GSL Gnu Scientific Library JsMath JavaScript implementation of LaTeX

These softwares are included in Sage:

IML Integer Matrix Library **IPvthon** Interactive Python shell LAPACK Fortan 77 linear algebra library Lcalc L-functions calculator Libgcrypt General purpose cryptographic library Libgpg-error Common error values for GnuPG components Linbox C++ linear algebra library Matplotlib Python plotting library Maxima computer algebra system Mercurial Revision control system MoinMoin Wiki

These softwares are included in Sage:

MPFI Multiple Precision Floating-point Interval library MPFR C library for multiple-precision floating-point computations **ECLib** Cremona's Programs for Elliptic curves NetworkX Graph theory NTL Number theory C++ library Numpy Numerical linear algebra OpenCDK Open Crypto Development Kit PALP A Package for Analyzing Lattice Polytopes PARI/GP Number theory calculator Pexpect Pseudo-tty control for Python PNG Bitmap image support

These softwares are included in Sage:

PolyBoRi Polynomials Over Boolean Rings PyCrypto | Python Cryptography Toolkit Python | Interpreted language Qd Quad-double/Double-double Computation Package Statistical Computing Readline Line-editing Python interface to R Rpy Scipy Python library for scientific computation Singular fast commutative and noncommutative algebra Scons Software construction tool SQLite Relation database

These softwares are included in Sage :

L-function calculator Sympow Symmetrica Representation theory Sympy Python library for symbolic computation Tachyon lightweight 3d ray tracer Termcap for writing portable text mode applications Twisted Python networking library Weave Tools for including C/C++ code within Python 7lib Data compression library ZODB Object-oriented database

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...and more!

> sage -gap

########		#####		##########		###	
############		#####		##########		####	
###############		#######		############		#####	
###	###########	####	####	#####	#####	:	#####
####	## #	####	#####	#####	#####	: #	#####
#####	#	#####	#####	#####	#####	##	#####
#####	:	#####	####	#####	######	###	#####
####		#####	#####	#######	#####	###	####
#####	######	####	####	#######	####	####	####
#####	######	#####	#####	######		####	####
#####	######	#####	#####	#####		######	######
####	# ####	#######	#######	#####		######	#######
####	## ####	#######	#######	#####		######	######
####	###########	########	########	#####			####
###	###########	#####	#####	#####			####
#	###########	#####	#####	#####			####
	########	#####	#####	#####			####

Information at: http://www.gap-system.org
Try '?help' for help. See also '?copyright' and '?authors'

Loading the library. Please be patient, this may take a while. GAP4, Version: 4.4.10 of 02-Oct-2007, x86_64-unknown-linux-gnu-gcc

gap>

> sage -singular

```
SINGULAR / Development
A Computer Algebra System for Polynomial Computations / version 3-1-0

by: G.-M. Greuel, G. Pfister, H. Schoenemann \ Mar 2009

FB Mathematik der Universitaet, D-67653 Kaiserslautern \
```

> sage -maxima

Maxima 5.16.3 http://maxima.sourceforge.net
Using Lisp ECL 9.4.1
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
The function bug_report() provides bug reporting information.
(%i1)

> sage -gp

GP/PARI CALCULATOR Version 2.3.3 (released) amd64 running linux (x86-64/GMP-4.2.1 kernel) 64-bit version compiled: Jul 10 2009, gcc-4.3.2 (Ubuntu 4.3.2-1ubuntu12) (readline v5.2 enabled, extended help available)

Copyright (C) 2000-2006 The PARI Group

PARI/GP is free software, covered by the GNU General Public License, and comes WITHOUT ANY WARRANTY WHATSOEVER.

Type ? for help, \q to quit. Type ?12 for how to get moral (and possibly technical) support.

parisize = 8000000, primelimit = 500000
?

> sage -R

R version 2.6.1 (2007-11-26) Copyright (C) 2007 The R Foundation for Statistical Computing ISBN 3-900051-07-0 $\,$

R is free software and comes with ABSOLUTELY NO WARRANTY. You are welcome to redistribute it under certain conditions. Type 'license()' or 'licence()' for distribution details.

Natural language support but running in an English locale

R is a collaborative project with many contributors. Type 'contributors()' for more information and 'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or 'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

Sage *combines* software.

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[This examples are from a presention of William Stein]

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Construct an elliptic curve using John Cremona's table:

```
sage: E = EllipticCurve('389a')
```

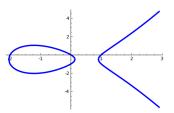
[This examples are from a presention of William Stein]

Construct an elliptic curve using John Cremona's table:

```
sage: E = EllipticCurve('389a')
```

Use *matplotlib* to plot it :

```
sage: plot(E,thickness=3)
```

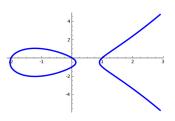


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```

Use *matplotlib* to plot it :



mwrank to do a 2-descent:

sage: E.mwrank()

Curve [0,1,1,-2,0]: Rank = 2

PARI to compute Fourier coefficients a_n :

```
sage: E.anlist(15)
[0, 1, -2, -2, 2, -3, 4, -5, 0, 1, 6, -4, -4, -3, 10, 6
```

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```
sage: E.anlist(15)
[0, 1, -2, -2, 2, -3, 4, -5, 0, 1, 6, -4, -4, -3, 10, 6]
```

lcalc to compute zeros in the critical strip of the L-series :

```
sage: E.lseries().zeros(5)
[0.000000000, 0.000000000, 2.87609907, 4.41689608, 5.79
```

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sympow to compute the modular degree :

```
sage: E.modular_degree()
40
```

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```

sympow to compute the modular degree :

```
sage: E.modular_degree()
40
```

Magma to compute the rank of the 3-selmer group :

```
sage: magma(E).ThreeSelmerGroup()
Abelian Group isomorphic to Z/3 + Z/3
Defined on 2 generators
```

```
Let's integrate \int \cos(x^2) dx:
```

```
sage: integrate(cos(x^2), x)
-1/8*((I + 1)*sqrt(2)*erf((1/2*I - 1/2)*sqrt(2)*x) +
(I - 1)*sqrt(2)*erf((1/2*I + 1/2)*sqrt(2)*x))*sqrt(pi)
```

Software used for this computation :

```
sage: from sage.misc.citation import get_systems
sage: get_systems("integrate(cos(x^2), x)")
['MPFI', 'ginac', 'GMP', 'Maxima']
```

"We implement all conversion routines, instead of expecting upstream to do it: we make them communicate with Sage, whether they want to or not. Resistence is futile."

—William Stein (started Sage)

Sage Optional Packages (spkg)

http://www.sagemath.org/packages/optional/

Fokko Ducloux's Coxeter 3 C++ library

```
sage -i coxeter3-1.1.spkg
```

Gnuplot.py is a Python package that interfaces to gnuplot

```
sage -i gnuplotpy-1.8.spkg
```

Ore Polynomials in Sage

```
http://www.risc.jku.at/research/combinat/software/ore_algebra/
```

```
sage -i ore_algebra-0.1.spkg
sage: from ore_algebra import *
```

SageManifolds: Differential geometry and tensor calculus http://sagemanifolds.obspm.fr/

```
sage -i http://sagemanifolds.obspm.fr/spkg/manifolds-0.2.spkg
sage: from manifolds.all import *
```



Sage Important Philosophy: Elements have parent

```
sage: m = matrix(ZZ, 3, [2,3,4,2,4,6,8,8,5])
sage: m.parent()
Full MatrixSpace of 3 by 3 dense matrices over Integer Ring
sage: m.echelon_form()
[2 0 1]
[0 1 2]
[0 0 3]
sage: m = matrix(QQ, 3, [2,3,4,2,4,6,8,8,5])
sage: m.parent()
Full MatrixSpace of 3 by 3 dense matrices over Rational Field
sage: m.echelon_form()
[1 0 0]
[0 \ 1 \ 0]
[0 0 1]
```

Sage uses <u>Python</u> as its programming language.

Sage uses *Python*

- ullet Sage pprox Python + a huge Python library
- Sage may be the first successful math software system to not invent its own new language just for mathematics.
- Tens of thousands of third party Python packages are immediately available for use with Sage!
- Easy to write and read :

$$\begin{array}{ll} \text{math}: & \left\{17x \middle| x \in \{0,1,...,9\} \text{ et } x \text{ est impair } \right\} \\ \text{python}: & \left[17*x \text{ for x in range(10) if x \% 2 == 1} \right] \\ \end{array}$$

Sage uses Python

« Google has made no secret of the fact they use Python a lot for a number of internal projects. Even knowing that, once I was an employee, I was amazed at how much Python code there actually is in the Google source code system. »

— Guido van Rossum

(Python creator)

Sage follows Best practices for Scientific Computing

Aruliah, D. A., C. Titus Brown, Neil P. Chue Hong, Matt Davis, Richard T. Guy, Steven H. D. Haddock, Katy Huff, et al. Best Practices for Scientific Computing.

September 30, 2012

http://arxiv.org/abs/1210.0530.

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Community

You are not alone!

Working together with internet

Development:

- http://groups.google.com/group/sage-devel
- http://trac.sagemath.org/sage_trac/
- http://wiki.sagemath.org/

Support:

- http://groups.google.com/group/sage-support
- http://ask.sagemath.org/questions/
- http://wiki.sagemath.org/

Other Google Group include: sage-combinat-devel, sage-release and sage-edu.

irc-channel

#sage-devel on freenode.net

Developer Map



There are currently 244+1+4+11 contributors in 165+1+2+9 different places from all around the world.

Sage Days

Sage Days are workshops aiming at

- fix bugs and develop new functionalities
- attract new users and developers

Dozen of workshops are organized every year all around the world.

Sage Days in 2010

- Sage Days 19 : Seattle, USA (January 2010)
- Sage Days 20 : Marseille, France (February 2010)
- Sage Days 20.25: Montreal, Canada (March 2010)
- ullet Sage Days 20.5: Toronto, Canada (May 2010)
- Sage Days 21 : Seattle, USA (June 2010)
- Sage-Combinat/Chevie : France (June 2010)
- Sage Days 22 : Berkeley, USA (July 2010)
- Sage Days 23 : Leiden, Netherlands (July 2010)
- Sage Days 23.5 : Kaiserslautern, Germany (July 2010)
- Sage Days 24: Linz, Austria (July 2010)
- Sage Days 25 : Mumbai, India (August 2010)
- Sage Days 25.5 : Montreal, Canada (September 2010)
- Sage Days 26: Seattle, USA (December 2010)



Sage Days in 2011

- Sage Days 27 : Seattle, USA (January 2011)
- Sage Days 28 : Orsay, France (January 2011)
- Sage Days 29: Seattle, USA (March 21-25, 2011)
- Sage Days 30: Halifax, Canada (May 2-6, 2011)
- Sage Days 31 : Seattle, USA (June 11-18, 2011)
- Sage Education Days 3: Seattle, USA (June 16-18, 2011)
- Sage Days 32: Seattle, USA (August 21-25, 2011)
- Sage Days 33: Women in Sage, Seattle (September 2011)
- Sage Days 34: Kaiserslautern, Germany (Sept. 2011)
- Sage Days 34.5 : Bobo Dioulasso, Burkina Faso (Nov. 2011)
- Sage Days 35: Warwick UK (December 2011)



Sage Days in 2012

- Sage Days 35.5 : Wenham, MA (January 2012)
- Sage-Combinat Days 35.75 : France (February 2012)
- Sage Days 36: UC San Diego, (February 2012)
- Review Days 2 : March 17-22, Seattle WA.
- Sage Days 36.5 : April 17-22, Seattle WA.
- Sage Days 37: 28 April 2012, Seoul, South Korea.
- Sage Days 38 : May 7-11 2012, Montréal, Canada.
- Bug Days 19 : May 24-29, Seattle WA.
- Sage Days 39 : May 26-27 2012, Fukuoka, Japan.
- Sage Days 41 : June 11-15, 2012, Seattle.
- Sage Edu Days 4 : June 13-15, 2012, Seattle.
- SAGE-Combinat Days 40: July 9-13, Minneapolis, MN.
- Sage Days 42: July 15-19, 2012, Seattle, WA.



Sage à Paris

http://wiki.sagemath.org/GroupeUtilisateursParis (meetings since November 2011) Montly meetings for *new users*, *users* and *developers* Goal: Learn from each other

Locations:

- Orsay
- Jussieu
- Chevaleret
- IRILL
- Télécom ParisTech

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Some functionalities

Interfaces:

- Command line
- Sage Notebook (local)
- Sage Notebook (internet)
- New: Sage in a web page (by Jason Grout, USA)

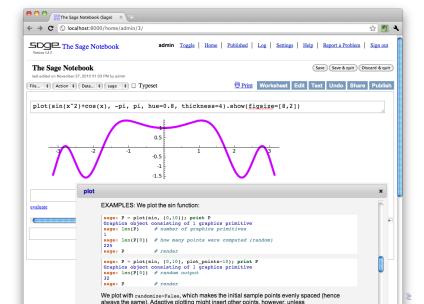
Interesting features:

- Sagetex (Dan Drake, Corée du Sud)
- Graph Editor (Radoslav Kirov, États-Unis)
- Interact and animations
- Cython : translates Python code \mapsto C code.

Command line interface

```
saliola@karkwa: ~
                                                                     File Edit View Terminal Tabs Help
saliola@karkwa:~$ sage
| SAGE Version 3.1.2, Release Date: 2008-09-19
| Type notebook() for the GUI, and license() for information.
sage: 3 * 17
51
sage: [ 17 * x for x in range(10) if x % 2 == 1 ]
[17, 51, 85, 119, 153]
sage: Partitions(4).list()
[[4], [3, 1], [2, 2], [2, 1, 1], [1, 1, 1, 1]]
sage:
```

Notebook interface (local)



Notebook interface (internet)

http://sagenb.org/



Welcome!

Sage is a different approach to mathematics software.

The Sage Notebook

With the Sage Notebook anyone can create, collaborate on, and publish interactive worksheets. In a worksheet, one can write code using Sage, Python, and other software included in Sage.

General and Advanced Pure and Applied Mathematics

Use Sage for studying calculus, elementary to very advanced number theory, cryptography, commutative algebra, group theory, graph theory, numerical and exact linear algebra, and more.

Use an Open Source Alternative

By using Sage you help to support a viable open source alternative to Magma, Maple, Mathematica, and MATLAB. Sage includes many high-quality open source math packages.

Use Most Mathematics Software from Within Sage

Sage makes it easy for you to use most mathematics software together. Sage includes GAP, GP/PARI, Maxima, and Singular, and dozens of other open packages.

Use a Mainstream Programming Language

You work with Sage using the highly regarded scripting language Python. You can write programs that combine serious mathematics with anything else.

Sign into the Sage Notebook v4.7 Username Password Remember me (sign in) Sign up for a new Sage Notebook account Browse published Sage worksheets (no login recurred)

Single Cell Server

http://www.liafa.univ-paris-diderot.fr/~labbe/blogue-index/http://www.sagemath.org/eval.html



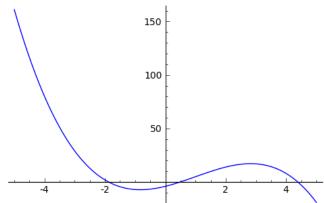
In this LATEXfile, I write

$$\space{2.5cm} \space{2.5cm} \space{2.5cm}$$

In this LATEXfile, I write

$$\space{2.5cm} \space{2.5cm} \space{2.5cm}$$

and this is replaced by :





In this LATEXfile:

```
\begin{sagesilent}
  sigma = Permutation([7,3,1,5,2,6,8,4])
  P, Q = sigma.robinson_schensted()
\end{sagesilent}

Let $\sigma = \sage{sigma}$. The
Robinson-Schensted-Knuth algorithm produces
the tableaux:
  \[\sage{P} \quad \sage{Q}\]
```

It got replaced with:

Let $\sigma = [7, 3, 1, 5, 2, 6, 8, 4]$. The Robinson-Schensted-Knuth algorithm produces the tableaux :

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Let $\sigma = [7, 3, 1, 5, 2, 6, 8, 4]$. The Robinson-Schensted-Knuth algorithm produces the tableaux :

This is done with the sagetex package for LATEX, written by Dan Drake. Of course, the package is included with Sage.

For more informations

http://sagemath.org



For more information

- http://sagemath.org
- http://sagemath.org/doc
- http://sagemath.org/help.html
- http://wiki.sagemath.org

Ressources in French:

- http://sagemath.org/fr
- http://sagemath.org/fr/html/tutorial

Le livre Calcul mathématique avec Sage :

• http://sagebook.gforge.inria.fr/

Let's start! Commençons!