

# Sage: free software for mathematics education and research

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# Outline

- 1 My history with math software
- 2 What is Sage?
- 3 What can Sage do?
- 4 Why use Sage?

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# Buy a copy for myself?

Not a practical solution.

# Sage: a much better solution

In October 2007 I started using Sage, and will tell you why it's an excellent choice for mathematics education and research.

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# What is Sage?

Sage is a “mathematics software system” composed of free and open source software.

Its mission is to create a viable, free, open source alternative to “the four M’s”: Magma, Maple, Mathematica and Matlab.

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# Under the hood

Sage is based on the well-known and powerful language Python and ties together many high-quality free software packages:

- GAP
- Maxima
- Singular
- Pari/GP
- R

Along with: SymPy, NumPy, Linbox, MPFR, MPFI, NTL, eclib, ATLAS, FLINT, lcalc, PolyBori, NetworkX, PyCrypto, cvxopt, matplotlib

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# Based on Python

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# Easy network access

The Sage notebook can be accessed with any modern web browser.



# Create fast compiled code

Sage includes Cython, which converts Python-like code to C code and compiles it into fast Python extensions.

# Integrate into $\text{\LaTeX}$ documents

Sage $\text{\TeX}$ , an optional add-on to Sage, allows you to integrate Sage computations into your  $\text{\LaTeX}$  documents.

# Experiment interactively

The “@interact” decorator allows easy interactive experiments.

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## Practical reasons

Anyone can use it for zero cost—no “dog whistle symphonies”, or retraining.

# Practical reasons

Sharing and collaboration is built into the Sage notebook.

## Practical reasons

Sage has interfaces to seamlessly interoperate with Maple, Mathematica, and Magma.

# Sage is Free software

“Free” means *freedom*, not just zero cost. Everyone has:

- the freedom to use Sage for any purpose;
- the freedom to study how it works, and adapt it to their needs;
- the freedom to redistribute copies of Sage;
- the freedom to improve Sage and release the improvements so the whole mathematical community benefits.

(Adapted from the Free Software Definition.)



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From Joachim Neubüser, who started GAP:

*You can read Sylow's Theorem and its proof [...] and then you can use Sylow's Theorem for the rest of your life free of charge, but for many computer algebra systems license fees have to be paid regularly [...]. You press buttons and you get answers in the same way as you get the bright pictures from your television set but you cannot control how they were made in either case.*

quote continues ...

*With this situation two of the most basic rules of conduct in mathematics are violated: in mathematics information is passed on free of charge and everything is laid open for checking. Not applying these rules to computer algebra systems that are made for mathematical research [...] means moving in a most undesirable direction. Most important: can we expect somebody to believe a result of a program that he is not allowed to see?*

# Free software is best for education

Training students to use commercial math software is like asking them to pay license fees to use the fundamental theorem of calculus.

It trains students to not ask questions about how things work.



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# Sage: free software for mathematics education and research

Visit `sagemath.org` or `sagenb.kaist.ac.kr`  
to learn about and use Sage.

Thank you